4 Momentum

4-1 Impulse and Momentum

Vocabulary **Momentum:** A measure of how difficult it is to stop a moving object.

$$\textbf{momentum} = \textbf{(mass)(velocity)} \quad \text{or} \quad p = mv$$

If the momentum of an object is changing, as it is when a force is exerted to start it or stop it, the change in momentum can be found by looking at the change in mass and velocity during the interval.

$$\textbf{change in momentum} = \textbf{change in [(mass)(velocity)]} \quad \text{or} \quad \Delta p = \Delta(mv)$$

For all the exercises in this book, assume that the mass of the object remains constant, and consider only the change in velocity, Δv, which is equal to $v_f - v_o$. Momentum is a vector quantity. Its direction is in the direction of the object's velocity.

The SI unit for momentum is the **kilogram·meter/second (kg·m/s).**

Vocabulary **Impulse:** The product of the force exerted on an object and the time interval during which it acts.

$$\textbf{impulse} = \textbf{(force)(elapsed time)} \quad \text{or} \quad J = F\Delta t$$

The SI unit for impulse is the **newton·second (N·s).**

The impulse given to an object is equal to the change in momentum of the object.

$$F\Delta t = m\Delta v$$

The same change in momentum may be the result of a large force exerted for a short time, or a small force exerted for a long time. In other words, impulse is the thing that you *do,* while change in momentum is the thing that you *see.*

The units for impulse and momentum are equivalent. Remember, $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$. Therefore, $1\ \text{N}\cdot\text{s} = 1\ \text{kg}\cdot\text{m/s}$.

Solved Examples

Example 1: Tiger Woods hits a 0.050-kg golf ball, giving it a speed of 75 m/s. What impulse does he impart to the ball?

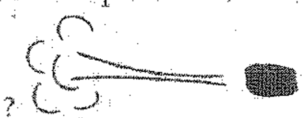
Solution: Because the impulse equals the change in momentum, you can reword this exercise to read, "What was the ball's change in momentum?" It is understood that the ball was initially at rest, so its initial speed was 0 m/s.

Given: $m = 0.050$ kg
$\Delta v = 75$ m/s

Unknown: $\Delta p = ?$
Original equation: $\Delta p = m\Delta v$

Solve: $\Delta p = (0.050 \text{ kg})(75 \text{ m/s}) = \mathbf{3.8\ kg \cdot m/s}$

Example 2: Wayne hits a stationary 0.12-kg hockey puck with a force that lasts for 1.0×10^{-2} s and makes the puck shoot across the ice with a speed of 20.0 m/s, scoring a goal for the team. With what force did Wayne hit the puck?



Given: $m = 0.12$ kg
$\Delta v = 20.0$ m/s
$\Delta t = 1.0 \times 10^{-2}$ s

Unknown: $F = ?$
Original equation: $F\Delta t = m\Delta v$

Solve: $F = \frac{m\Delta v}{\Delta t} = \frac{(0.12 \text{ kg})(20.0 \text{ m/s})}{1.0 \times 10^{-2} \text{ s}} = 240 \text{ kg}\cdot\text{m/s}^2 = \mathbf{240\ N}$

Example 3: A tennis ball traveling at 10.0 m/s is returned by Venus Williams. It leaves her racket with a speed of 36.0 m/s in the opposite direction from which it came. a) What is the change in momentum of the tennis ball? b) If the 0.060-kg ball is in contact with the racket for 0.020 s, with what average force has Venus hit the ball?

Solution: In this exercise, the tennis ball is coming toward Venus with a speed of 10.0 m/s in one direction, but she hits it back with a speed of 36.0 m/s in the opposite direction. Therefore, you must think about velocity vectors and call one direction positive and the opposite direction negative.

a. *Given:* $v_o = -10.0$ m/s
$v_f = 36.0$ m/s
$m = 0.060$ kg

Unknown: $\Delta p = ?$
Original equation: $\Delta p = m\Delta v = m(v_f - v_o)$

Solve: $\Delta p = m(v_f - v_o) = (0.060 \text{ kg})[36.0 \text{ m/s} - (-10.0 \text{ m/s})] = \mathbf{2.8\ kg \cdot m/s}$

b. *Given:* $m = 0.060$ kg
$\Delta v = 46.0$ m/s
$\Delta t = 0.020$ s

Unknown: $F = ?$
Original equation: $F\Delta t = m\Delta v$

Solve: $F = \frac{m\Delta v}{\Delta t} = \frac{(0.060 \text{ kg})(46.0 \text{ m/s})}{(0.020 \text{ s})} = \mathbf{140\ N}$

Example 4: To demonstrate his new high-speed camera, Flash performs an experiment in the physics lab in which he shoots a pellet gun at a pumpkin to record the moment of impact on film. The 1.0-g pellet travels at 100. m/s until it embeds itself 2.0 cm into the pumpkin. What average force does the pumpkin exert to stop the pellet?

Solution: First, convert g to kg and cm to m.

$$1.0 \text{ g} = 0.0010 \text{ kg} \qquad 2.0 \text{ cm} = 0.020 \text{ m}$$

Before you can solve for the force in the exercise, you must first know how long the force is being exerted. Remember, in order to find the time, you must use the average velocity, v_{av}.

$$v_{av} = \frac{v_f + v_o}{2} = \frac{0 \text{ m/s} + 100. \text{ m/s}}{2} = 50.0 \text{ m/s}$$

Given: $v = 50.0$ m/s
$\Delta d = 0.020$ m

Unknown: $\Delta t = ?$
Original equation: $\Delta d = v\Delta t$

$$\textit{Solve: } \Delta t = \frac{\Delta d}{v} = \frac{0.020 \text{ m}}{50.0 \text{ m/s}} = 0.00040 \text{ s}$$

Now we can solve for the force the pumpkin exerts to stop the pellet.

Given: $m = 0.0010$ kg
$\Delta v = 100.$ m/s
$\Delta t = 0.0040$ s

Unknown: $F = ?$
Original equation: $F\Delta t = m\Delta v$

$$\textit{Solve: } F = \frac{m\Delta v}{\Delta t} = \frac{(0.0010 \text{ kg})(100. \text{ m/s})}{(0.00040 \text{ s})} = \mathbf{250 \text{ N}}$$

Practice Exercises

Exercise 1: On April 15, 1912, the luxury cruiseliner *Titanic* sank after running into an iceberg. a) What momentum would the 4.23×10^8-kg ship have imparted to the iceberg if it had hit the iceberg head-on with a speed of 11.6 m/s? (Actually, the impact was a glancing blow.) b) If the captain of the ship had seen the iceberg a kilometer ahead and had tried to slow down, why would this have been a futile effort?

$m = 4.23 \times 10^8$ kg
$v = 11.6$ m/s

$p = mv$
$p = (4.23 \times 10^8 \text{ kg})(11.6 \text{ m/s})$

4.91×10^9 kgm/s

Answer: a. ______________

Answer: b. No, it takes to long to turn or stop a ship.

Exercise 2: Auto companies frequently test the safety of automobiles by putting them through crash tests to observe the integrity of the passenger compartment. If a 1000.-kg car is sent toward a cement wall with a speed of 14 m/s and the impact brings it to a stop in 8.00×10^{-2} s, with what average force is it brought to rest?

M=1000 kg
V=14 m/s
t=8.00x10⁻² s
F=?

$Ft = mv$ $\quad F = \frac{mv}{t}$

$F = \frac{(1000\,kg)(14\,m/s)}{8.00 \times 10^{-2}\,s}$

1.8×10^5 N

Answer: ______________

Exercise 3: Rhonda, who has a mass of 60.0 kg, is riding at 25.0 m/s in her sports car when she must suddenly slam on the brakes to avoid hitting a dog crossing the road. She is wearing her seatbelt, which brings her body to a stop in 0.400 s. a) What average force did the seatbelt exert on her? b) If she had not been wearing her seatbelt, and the windshield had stopped her head in 1.0×10^{-3} s, what average force would the windshield have exerted on her? c) How many times greater is the stopping force of the windshield than the seatbelt?

m=60.0 kg
V=25.0 m/s
t=.400 s
F=?

$t_2 = 1.0 \times 10^{-3}$ s
F=?

$Ft = mv$ $\quad F = \frac{mv}{t}$

$F = \frac{(60.0\,kg)(25.0\,m/s)}{.400\,s}$ $\quad$ 3750 N

$F = \frac{(60.0\,kg)(25.0\,m/s)}{1.0 \times 10^{-3}\,s}$ $\quad$ 1.5×10^6 N

Answer: a. 3750 N

Answer: b. ~~1.5×10^6 N~~

Answer: c. 400x

$\frac{1.5 \times 10^6\,N}{3750\,N} = 400$

Exercise 4: If 270 million people in the United States jumped up in the air simultaneously, pushing off Earth with an average force of 800. N each for a time of 0.10 s, what would happen to the 5.98×10^{24} kg Earth? Show a calculation that justifies your answer.

multiply the # of people by the force because each person makes this force.

270,000,000
F=800. N
t=.10 s
m=5.98x10²⁴ kg
V=?

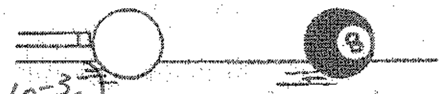
$Ft = mv$ $\quad V = \frac{Ft}{m}$

$V = \frac{(270{,}000{,}000)(800.\,N)(.10\,s)}{5.98 \times 10^{24}}$

Answer: ______________ 3.61×10^{-15} m/s

very slow

Exercise 5: In Sharkey's Billiard Academy, Maurice is waiting to make his last shot. He notices that the cue ball is lined up for a perfect head-on collision, as shown. Each of the balls has a mass of 0.0800 kg and the cue ball comes to a complete stop upon making contact with the 8 ball. Suppose Maurice hits the cue ball by exerting a force of 180. N for 5.00×10^{-3} s, and it knocks head-on into the 8 ball. Calculate the resulting velocity of the 8 ball.



$m = .0800\,kg$
$F = 180.\,N$
$t = 5.00 \times 10^{-3}\,s$
$v = ?$

$Ft = mv$

$v = \frac{Ft}{m}$

$v = \frac{(180.\,N)(5 \times 10^{-3}\,s)}{.0800\,kg}$

$11.3\,m/s$

Answer: ______________________

Exercise 6: During an autumn storm, a 0.012-kg hail stone traveling at 20.0 m/s made a 0.20-cm-deep dent in the hood of Darnell's new car. What average force did the car exert to stop the damaging hail stone?

change to m

$m = .012\,kg$
$v_i = 20.0\,m/s$
$d = .0020\,m$
$F = ?$
$v_f = 0$

Find average velocity

$\frac{v_i + v_f}{2} = \frac{20.0\,m/s + 0}{2} = 10.0\,m/s$

$v = \frac{d}{t}$

$t = \frac{d}{v} = \frac{.0020\,m}{10.0\,m/s} = .00020\,s$

t = time the force is stopping the hail.

$Ft = mv$

$F = \frac{mv}{t}$

$F = \frac{(.012\,kg)(20.0\,m/s)}{.00020\,s}$

$1200\,N$

Answer: ______________________

4-2 Conservation of Momentum

According to the **law of conservation of momentum,** the total momentum in a system remains the same if no external forces act on the system. Consider the two types of collisions that can occur.

Vocabulary **Elastic collision:** A collision in which objects collide and bounce apart with no energy loss.

In an elastic collision, because momentum is conserved, the *mv* before a collision for each of the two objects must equal the *mv* after the collision for each of the two objects. This is written as

$$m_1v_{1o} + m_2v_{2o} = m_1v_{1f} + m_2v_{2f}$$

The subscripts 1 and 2 refer to objects 1 and 2, respectively.

Vocabulary **Inelastic collision:** A collision in which objects collide and some mechanical energy is transformed into heat energy.

A common kind of inelastic collision is one in which the colliding objects stick together, or start out stuck together and then separate. However, in an inelastic collision the objects need not remain stuck together but may instead deform in some way.

Because momentum is also conserved in an inelastic collision, the mv before the collision for each of the two objects must equal the mv after the collision for each of the two objects. When objects are stuck together after the collision (assuming mass does not change), this equation becomes

$$m_1v_{1o} + m_2v_{2o} = (m_1 + m_2)v_f$$

where v_f is the combined final velocity of the two objects.

Solved Examples

Example 5: Tubby and his twin brother Chubby have a combined mass of 200.0 kg and are zooming along in a 100.0-kg amusement park bumper car at 10.0 m/s. They bump Melinda's car, which is sitting still. Melinda has a mass of 25.0 kg. After the elastic collision, the twins continue ahead with a speed of 4.12 m/s. How fast is Melinda's car bumped across the floor?

Solution: Notice that you must add the mass of the bumper car to the mass of the riders.

Given: $m_1 = 300.0$ kg
$m_2 = 125.0$ kg
$v_{1o} = 10.0$ m/s
$v_{2o} = 0$ m/s
$v_{1f} = 4.12$ m/s

Unknown: $v_{2f} = ?$
Original equation:
$m_1v_{1o} + m_2v_{2o} = m_1v_{1f} + m_2v_{2f}$

Solve:
$$v_{2f} = \frac{m_1v_{1o} + m_2v_{2o} - m_1v_{1f}}{\mathrm{m}_2}$$

$$= \frac{(300.0\text{ kg})(10.0\text{ m/s}) + (125.0\text{ kg})(0\text{ m/s}) - (300.0\text{ kg})(4.12\text{ m/s})}{125.0\text{ kg}}$$

$$= \frac{3000\text{ kg}\cdot\text{m/s} + 0\text{ kg}\cdot\text{m/s} - 1236\text{ kg}\cdot\text{m/s}}{125.0\text{ kg}} = \frac{1764\text{ kg}\cdot\text{m/s}}{125.0\text{ kg}}$$

$$= \mathbf{14.1\ m/s}$$

Example 6: Sometimes the curiosity factor at the scene of a car accident is so great that it actually produces secondary accidents as a result, while people watch to see what is going on. If an 800.-kg sports car slows to 13.0 m/s to check out an accident scene and the 1200.-kg pick-up truck behind him continues traveling at 25.0 m/s, with what velocity will the two move if they lock bumpers after a rear-end collision?

Solution: Since the two vehicles lock bumpers, both objects have the same final velocity.

Given: $m_1 = 800.\ \text{kg}$
$m_2 = 1200.\ \text{kg}$
$v_{1o} = 13.0\ \text{m/s}$
$v_{2o} = 25.0\ \text{m/s}$

Unknown: $v_f = ?$
Original equation:
$m_1v_{1o} + m_2v_{2o} = (m_1 + m_2)v_f$

$$\textit{Solve: } v_f = \frac{m_1v_{1o} + m_2v_{2o}}{(m_1 + m_2)} = \frac{(800.\ \text{kg})(13.0\ \text{m/s}) + (1200.\ \text{kg})(25.0\ \text{m/s})}{(800.\ \text{kg} + 1200.\ \text{kg})}$$

$$= \frac{10\,400\ \text{kg}\cdot\text{m/s} + 30\,000\ \text{kg}\cdot\text{m/s}}{2000.\ \text{kg}} = \mathbf{20.2\ m/s\ forward}$$

Example 7: Charlotte, a 65.0-kg skin diver, shoots a 2.0-kg spear with a speed of 15 m/s at a fish who darts quickly away without getting hit. How fast does Charlotte move backwards when the spear is shot?

Solution: To start, Charlotte and the spear are together and both are at rest.

Given: $m_1 = 65.0\ \text{kg}$
$m_2 = 2.0\ \text{kg}$
$v_o = 0\ \text{m/s}$
$v_{2f} = 15.0\ \text{m/s}$

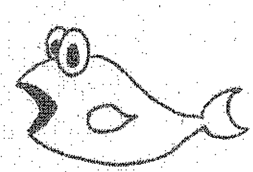
Unknown: $v_{1f} = ?$
Original equation:
$(m_1 + m_2)v_o = m_1v_{1f} + m_2v_{2f}$

$$\textit{Solve: } v_{1f} = \frac{(m_1 + m_2)v_o - m_2v_{2f}}{m_1}$$

$$= \frac{(65.0\ \text{kg} + 2.0\ \text{kg})(0\ \text{m/s}) - (2.0\ \text{kg})(15\ \text{m/s})}{65.0\ \text{kg}}$$

$$= \frac{-30\ \text{kg}\cdot\text{m/s}}{65.0\ \text{kg}} = \mathbf{-0.46\ m/s}$$

Remember, the minus sign here is indicating direction. Therefore, Charlotte would travel with a speed of 0.46 m/s in a direction opposite to that of the spear.



Practice Exercises

Exercise 7: Jamal is at the state fair playing some of the games. At one booth he throws a 0.50-kg ball forward with a velocity of 21.0 m/s in order to hit a 0.20-kg bottle sitting on a shelf, and when he makes contact the bottle goes flying forward at 30.0 m/s. a) What is the velocity of the ball after it hits the bottle? b) If the bottle were more massive, how would this affect the final velocity of the ball?

$\boxed{.50kg} + \boxed{.20kg} = \boxed{.50kg} + \boxed{.20kg}$

21.0m/s 0m/s ? 30.0m/s

$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

$(.50kg)(21.0m/s) + (.20kg)(0) = (.50kg)v_{1f} + (.20kg)(30.0m/s)$

$10.5kgm/s + 0 = (.50kg)(v_{1f}) + 6.0kgm/s$

-6.0

Answer: a. ______________________

$4.5kgm/s = (.50kg)(v_{1f})$

Answer: b. increase

9.0 m/s

Exercise 8: Jeanne rolls a 7.0-kg bowling ball down the alley for the league championship. One pin is still standing, and Jeanne hits it head-on with a velocity of 9.0 m/s. The 2.0-kg pin acquires a forward velocity of 14.0 m/s. What is the new velocity of the bowling ball?

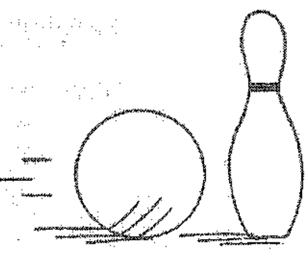
$\boxed{7.0kg} + \boxed{2.0kg} = \boxed{7.0kg} + \boxed{2.0kg}$

9.0m/s 0 ? 14.0m/s

$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

$(7.0kg)(9.0m/s) + 0 = (7.0kg)v_{1f} + (2.0kg)(14.0m/s)$

$63kgm/s + 0 = (7.0kg)v_{1f} + 28kgm/s$



$35kgm/s$ Answer: ~~(7.0kg)v1f~~ $v_{1f} = 5.0m/s$

Exercise 9: Running at 2.0 m/s, Bruce, the 45.0-kg quarterback, collides with Biff, the 90.0-kg tackle, who is traveling at 7.0 m/s in the other direction. Upon collision, Biff continues to travel forward at 1.0 m/s. How fast is Bruce knocked backwards?

Different Direction, one has to be negative.

If you make the other direction negative you get a positive answer.

$\boxed{45.0kg} + \boxed{90.0kg} = \boxed{45.0kg} + \boxed{90.0kg}$

2.0m/s -7.0m/s ? -1.0m/s

$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

$(45.0kg)(2.0m/s) + (90.0kg)(-7.0m/s) = (45.0kg)v_{1f} + (90.0kg)(-1.0m/s)$

$90.kgm/s + -630kgm/s = (45.0kg)v_{1f} + -90.0kg$

Answer: ______________________

$-540.kgm/s = (45.0kg)v_{1f} + -90.0kg$

$-450kgm/s = (45.0kg)v_{1f}$

$-10.m/s$

Exercise 10: Anthony and Sissy are participating in the "Roll-a-Rama" rollerskating dance championship. While 75.0-kg Anthony rollerskates backwards at 3.0 m/s, 60.0-kg Sissy jumps into his arms with a velocity of 5.0 m/s in the same direction. a) How fast does the pair roll backwards together? b) If Anthony is skating toward Sissy when she jumps, would their combined final velocity be larger or smaller than your answer to part a? Why?

$\boxed{60.0\text{ kg}}$ + $\boxed{75.0\text{ kg}}$ = $\boxed{60.0\text{kg} \mid 75.0\text{kg}}$

5.0 m/s → 3.0 m/s → ?

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f$$

$$(60.0\text{ kg})(5.0\text{ m/s}) + (75.0\text{ kg})(3.0\text{ m/s}) = (60.0\text{ kg} + 75.0\text{ kg})v_f$$

$$300.\text{ kg m/s} + 225\text{ kg m/s} = (135\text{ kg})v_f$$

$$525\text{ kg m/s} = (135\text{ kg})v_f$$

$$v_f = 3.9\text{ m/s}$$

Answer: a. ______________

Answer: b. Smaller

Exercise 11: To test the strength of a retainment wall designed to protect a nuclear reactor, a rocket-propelled F-4 Phantom jet aircraft was crashed head-on into a concrete barrier at high speed in Sandia, New Mexico on April 19, 1988. The F-4 phantom had a mass of 19100 kg, while the retainment wall's mass was 469000 kg. The wall sat on a cushion of air that allowed it to move during impact. If the wall and F-4 moved together at 8.41 m/s during the collision, what was the initial speed of the F-4 Phantom?

$\boxed{19100\text{ kg}}$ + $\boxed{469000\text{ kg}}$ = $\boxed{19100\text{ kg} \mid 469000\text{ kg}}$

? 0 8.41 m/s

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f$$

$$(19100\text{ kg})v_{1i} + 0 = (19100\text{ kg} + 469000\text{ kg})\,8.41\text{ m/s}$$

$$(19100\text{ kg})v_{1i} = 4.10 \times 10^6\text{ kg m/s}$$

$$v_{1i} = 215\text{ m/s}$$

Answer: ______________

Exercise 12: Valentina, the Russian Cosmonaut, goes outside her ship for a spacewalk, but when she is floating 15 m from the ship, her tether catches on a sharp piece of metal and is severed. Valentina tosses her 2.0-kg camera away from the spaceship with a speed of 12 m/s. a) How fast will Valentina, whose mass is now 68 kg, travel toward the spaceship? b) Assuming the spaceship remains at rest with respect to Valentina, how long will it take her to reach the ship?

If you set the other direction negative you would get a positive answer.

$$\boxed{68kg \mid 2.0kg} = \boxed{68kg} + \boxed{2.0kg}$$

0 ? 12m/s

$$(m_1 + m_2)v_i = m_1 v_{1f} + m_2 v_{2f}$$

$$0 = (68kg)v_{1f} + (2.0kg)(12m/s)$$

$$0 = (68kg)v_{1f} + (24\ kg\,m/s)$$

$$-24\ kg\,m/s$$

$$v_{1f} = -.35m/s$$

Answer: a. ______________

Answer: b. ______________

$$-24\ kg\,m/s = (68kg)v_{1f}$$

Exercise 13: A 620.-kg moose stands in the middle of the railroad tracks, frozen by the lights of an oncoming 10 000.-kg locomotive that is traveling at 10.0 m/s. The engineer sees the moose but is unable to stop the train in time and the moose rides down the track sitting on the cowcatcher. What is the new combined velocity of the locomotive and the moose?

$$\boxed{620.kg} + \boxed{10,000.kg} = \boxed{620.kg \mid 10,000kg}$$

0m/s 10.0m/s

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)v_f$$

$$0 + (10,000kg)(10.0m/s) = (620.kg + 10,000kg)v_f$$

$$100000kg\,m/s = (10,620\ kg)v_f$$

Answer: ______________

$$v_f = 9.42m/s$$

Exercise 14: Lee is rolling along on her 4.0-kg skateboard with a constant speed of 3.0 m/s when she jumps off the back and continues forward with a velocity of 2.0 m/s relative to the ground. This causes the skateboard to go flying forward with a speed of 15.5 m/s relative to the ground. What is Lee's mass?

Never mind omit

$$\boxed{4.0kg \mid ?} = \boxed{4.0kg} + \boxed{?}$$

3.0m/s 15.5m/s 2.0m/s

~~12 kg m/s~~

~~75.0 kg m/s~~

$$(m_1 + m_2)v_f = m_1 v_{1f} + m_2 v_{2f}$$

$$- m_2 v_{2f}$$

Answer: ______________

$$m_2 = \frac{m_1 v_{1f}}{[(m_1+1)v_f - v_{2f}]}$$

It is a factor problem we are

$$(m_1 + m_2)v_f - m_2 v_{2f} = m_1 v_{1f}$$

not going to do.

$$m_2 (m_1 + 1)v_f - v_{2f} = m_1 v_{1f}$$

$$m_2 = \frac{(4.0kg)(3.0m/s)}{([(4.0kg + 1)15.5m/s) - 2.0m/s)}$$

I forgot that we do not do this problem.