PRE-AP PRE-CALCULUS—PARAMETRIC EQUATIONS NOTES—NAME: _____

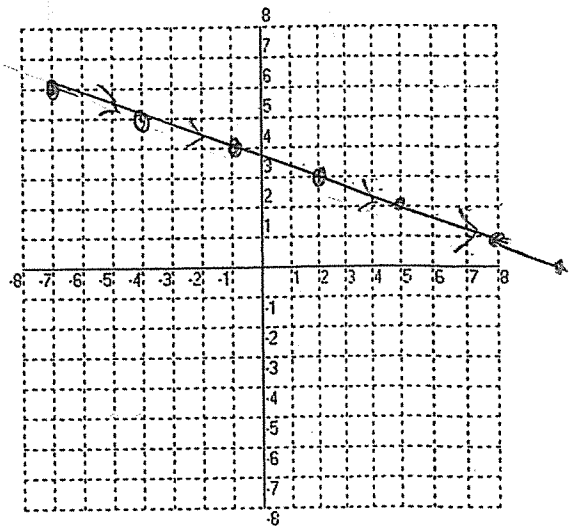
Given the parametric equations $\begin{cases} x = 3t + 2 \\ y = -t + 3 \end{cases}$,

a. Complete the table of values to the right.

b. Use the table of values to graph the parametric equations.

c. What is the graph of the parametric equations?

t	x	y
-3	-7	6
-2	-4	5
-1	-1	4
0	2	3
1	5	2
2	8	1
3	11	0



d. Write a function to represent the graph of the parametric equations.

$y = -1/3x + 11/3$

e. Write the corresponding rectangular equation by eliminating the parameter

$x - 2 = 3t$

$\frac{x-2}{3} = t$

$t = 3 - y$

$\frac{x-2}{3} = 3 - y$

$x - 2 = 9 - 3y$

$x - 11 = -3y$

$y = -\frac{1}{3}x + \frac{11}{3}$

f. What is the x-intercept of the graph? For what value of t does the graph cross the x-axis?

$(11, 0)$ $\quad t = 3$

g. What is the y-intercept of the graph? For what value of t does the graph cross the y-axis?

$(0, 11/3)$

$t = \frac{-2}{3}$

$\frac{11}{3} = -t + 3$

$\frac{2}{3} = -t$

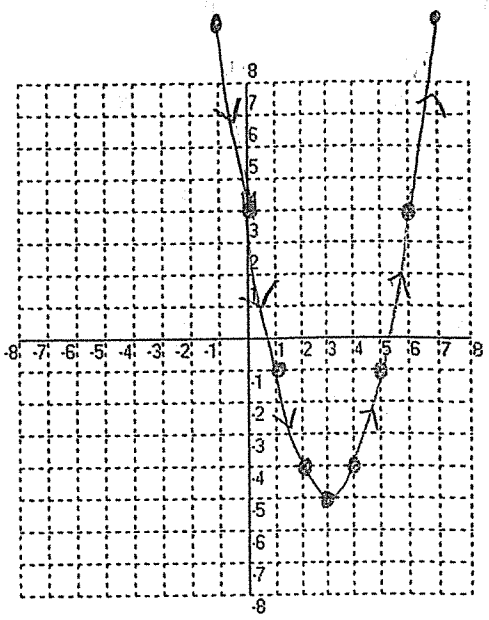
2. Given the parametric equations $\begin{cases} x = t+3 \\ y = t^2 - 5 \end{cases}$

a. Complete the table of values to the right.

b. Use the table of values to graph the parametric equations.

c. What is the graph of the parametric equations?

t	x	y
-4	-1	11
-3	0	4
-2	1	-1
-1	2	-4
0	3	-5
1	4	-4
2	5	-1
3	6	4
4	7	11



d. Write a function to represent the graph of the parametric equations.

$y = (x-3)^2 - 5$

e. Write the corresponding rectangular equation by eliminating the parameter

$t = x - 3$

$t^2 = y + 5$

$t = \sqrt{y+5}$

$x - 3 = \sqrt{y+5}$

$(x-3)^2 = y + 5$

$y = (x-3)^2 - 5$

f. What is the x-intercept of the graph? For what value of t does the graph cross the x-axis?

$0 = (x-3)^2 - 5$

$5 = (x-3)^2$

$x - 3 = \pm\sqrt{5}$

$x = 3 \pm \sqrt{5}$

$(3 \pm \sqrt{5}, 0)$

$3 \pm \sqrt{5} = t + 3$

$t = \pm\sqrt{5}$

g. What is the y-intercept of the graph? For what value of t does the graph cross the y-axis?

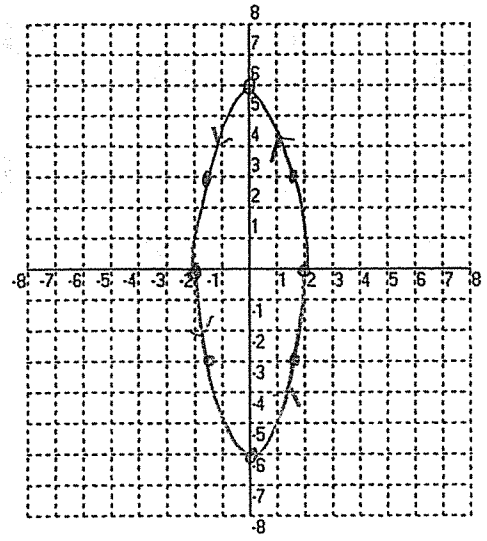
$(0, 4)$ $t = -3$

3. Given the parametric equations $\begin{cases} x = 2\cos\theta \\ y = 6\sin\theta \end{cases}$

a. Complete the table of values to the right.

b. Use the table of values to graph the parametric equations.

θ	x	y	θ	x	y
0°	2	0	210°	$-\sqrt{3}$	-3
30°	$\sqrt{3}$	3	225°	$-\sqrt{2}$	$-3\sqrt{2}$
45°	$\sqrt{2}$	$3\sqrt{2}$	240°	-1	$-3\sqrt{3}$
60°	1	$3\sqrt{3}$	270°	0	-6
90°	0	6	300°	1	$-3\sqrt{3}$
120°	-1	$3\sqrt{3}$	315°	$\sqrt{2}$	$-3\sqrt{2}$
135°	$-\sqrt{2}$	$3\sqrt{2}$	330°	$\sqrt{3}$	-3
150°	$-\sqrt{3}$	3	360°	2	0
180°	-2	0	390°	$\sqrt{3}$	3



c. What is the graph of the parametric equations?

ellipse

d. Write a function to represent the graph of the parametric equations.

$\frac{x^2}{4} + \frac{y^2}{36} = 1$

e. Write the corresponding rectangular equation by eliminating the parameter

$\frac{x}{2} = \cos\theta \qquad \frac{y}{6} = \sin\theta \qquad \cos^2\theta + \sin^2\theta = 1$

$\frac{x^2}{4} + \frac{y^2}{36} = 1$

f. What is the x-intercept of the graph? For what value of θ does the graph cross the x-axis?

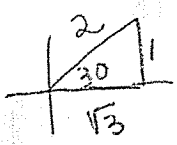
$(2,0)$ $\quad \theta = 0°, 360°$

$(-2,0)$ $\quad \theta = 180°$

g. What is the y-intercept of the graph? For what value of θ does the graph cross the y-axis?

$(0,6)$ $\quad \theta = 90°$

$(0,-6)$ $\quad \theta = 270°$

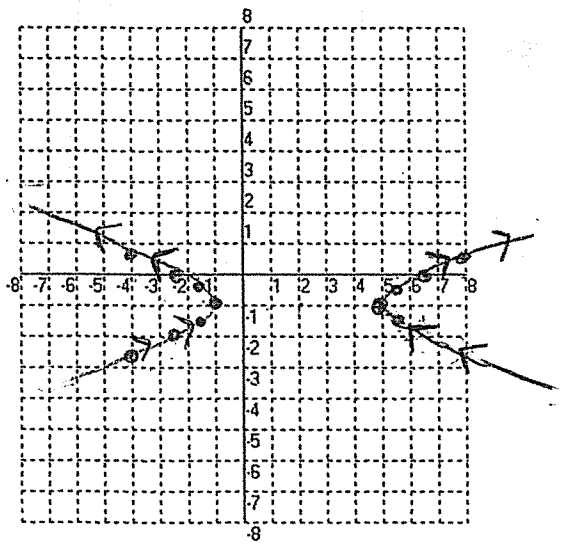


4. Given the parametric equations $\begin{cases} x = 2 + 3\sec\theta \\ y = -1 + \tan\theta \end{cases}$

a. Complete the table of values to the right.

b. Use the table of values to graph the parametric equations.

θ	x	y	θ	x	y
0°	5	−1	210°	−1.5	−.4
30°	5.5	−.4	225°	−2.2	0
45°	6.3	0	240°	−4	.73
60°	8	.73	270°	error	
90°	error		300°	8	−2.7
120°	−4	−2.7	315°	6.2	−2
135°	−2.2	−2	330°	5.5	−1.6
150°	−1.5	−1.6	360°	5	−1
180°	−1	−1	390°	5.5	−.4



c. What is the graph of the parametric equations?

hyperbola

d. Write a function to represent the graph of the parametric equations.

$$\frac{(x-2)^2}{9} - \frac{(y+1)^2}{1} = 1$$

e. Write the corresponding rectangular equation by eliminating the parameter.

$\frac{x-2}{3} = \sec\theta$

$y+1 = \tan\theta$

$\sec^2\theta - \tan^2\theta = 1$

$\frac{(x-2)^2}{9} - \frac{(y+1)^2}{1} = 1$

f. What is the x-intercept of the graph? For what value of θ does the graph cross the x-axis?

$0 = -1 + \tan\theta$

$\tan\theta = 1$

$\theta = 45°, 225°$

g. What is the y-intercept of the graph? For what value of θ does the graph cross the y-axis?

$0 = 2 + 3\sec\theta$

$-\frac{2}{3} = \sec\theta$

no y-intercept

5. Write the corresponding rectangular equation by eliminating the parameter.

a. $x = 3t - 1$, $y = 2t + 1$

$y = \frac{2x}{3} + \frac{2}{3} + 1$

$x + 1 = 3t$

$\boxed{y = \frac{2x}{3} + \frac{5}{3}}$

$\frac{x}{3} + \frac{1}{3} = t$

$y = 2\left(\frac{x}{3} + \frac{1}{3}\right) + 1$

b. $x = 3 - 2t$, $y = 2 + 3t$

$y = 2 - 3/2x + 9/2$

$\boxed{y = -3/2x + 13/2}$

$x - 3 = -2t$

$t = -1/2x + 3/2$

$y = 2 + 3(-1/2x + 3/2)$

c. $x = t + 1$, $y = t^2$

$x - 1 = t$

$y = (x-1)^2$

$\boxed{y = x^2 - 2x + 1}$

d. $x = \sqrt[3]{t}$, $y = 1 - t$

$x^3 = t$

$\boxed{y = 1 - x^3}$

e. $x = t^3$, $y = \frac{t^2}{2}$

$t = \sqrt[3]{x}$

$y = \frac{(\sqrt[3]{x})^2}{2}$

$\boxed{y = \frac{x^{2/3}}{2}}$

f. $x = t - 1$, $y = \frac{t}{t-1}$

$x + 1 = t$

$y = \frac{x+1}{x+1-1}$

$\boxed{y = \frac{x+1}{x}}$

g. $x = 1 + \frac{1}{t}$, $y = t - 1$

$y + 1 = t$

$\boxed{x = 1 + \frac{1}{y+1}}$

$x - 1 = \frac{1}{t}$

$t(x-1) = 1$

$t = \frac{1}{x-1}$

$\boxed{y = \frac{1}{x-1} - 1}$

h. $x = \sec\theta$, $y = \cos\theta$

$x = \frac{1}{\cos\theta}$

$\boxed{y = \frac{1}{x}}$

$x = \frac{1}{y}$

$xy = 1$

i. $x = \tan^2\theta$
$y = \sec^2\theta$

$\sec^2\theta - \tan^2\theta = 1$

$\boxed{y - x = 1}$

$x = 4 + 2\cos\theta$
$y = -1 + \sin\theta$

$\frac{x-4}{2} = \cos\theta \qquad y+1 = \sin\theta$

$\boxed{\frac{(x-4)^2}{4} + \frac{(y+1)^2}{1} = 1}$

j. $x = 3\cos\theta$
$y = 3\sin\theta$

$\frac{x}{3} = \cos\theta \qquad \frac{y}{3} = \sin\theta$

$\boxed{\frac{x^2}{9} + \frac{y^2}{9} = 1}$

l. $x = -2 + 6\cos\theta$
$y = 3 + 4\sin\theta$

$\frac{x+2}{6} = \cos\theta \qquad \frac{y-3}{4} = \sin\theta$

$\frac{(x+2)^2}{36} + \frac{(y-3)^2}{16} = 1$

k. $x = \cos\theta$
$y = 3\sin\theta$

$\frac{x}{1} = \cos\theta$

$\frac{y}{3} = \sin\theta$

$\boxed{\frac{x^2}{1} + \frac{y^2}{9} = 1}$

m. $x = 5 + 2\cos\theta$
$y = -3 + 2\sin\theta$

$\frac{x-5}{2} = \cos\theta$

$\frac{y+3}{2} = \sin\theta$

$\boxed{\frac{(x-5)^2}{4} + \frac{(y+3)^2}{4} = 1}$

6. The rectangular form of an ellipse is $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$. The corresponding parametric form of an ellipse is $\begin{cases} x=h+a\cos\theta \\ y=k+b\sin\theta \end{cases}$. Write each of the following in parametric form.

a. $\frac{x^2}{9}+\frac{y^2}{81}=1$ $a=3$ $b=9$

$x=3\cos\theta$
$y=9\sin\theta$

V(2,3)

b. $\frac{(x-2)^2}{16}+\frac{(y-3)^2}{1}=1$ $a=4$ $b=1$

$x=2+4\cos\theta$
$y=3+\sin\theta$

c. $(x+4)^2+\frac{(y+1)^2}{9}=1$ V(-4,-1) $a=1$ $b=3$

$x=-4+\cos\theta$
$y=-1+3\sin\theta$

d. $x^2+4y^2-2x-16y+1=0$

$x^2-2x+1+4(y^2-4y+4)=-1+1+16$

$(x-1)^2+4(y-2)^2=16$

$\frac{(x-1)^2}{16}+\frac{(y-2)^2}{4}=1$

V(1,2) $a=4$ $b=2$

$x=1+4\cos\theta$
$y=2+2\sin\theta$

7. The rectangular form of a hyperbola is $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$. The corresponding parametric form of a hyperbola is $\begin{cases} x=h+a\sec\theta \\ y=k+b\tan\theta \end{cases}$. Write each of the following in parametric form.

a. $\frac{x^2}{25}-\frac{y^2}{9}=1$ $a=5$ $b=3$

$x=5\sec\theta$
$y=3\tan\theta$

b. $(y-2)^2-(x-4)^2=1$ (4,2) $a=1$ $b=1$

$x=4+\tan\theta$
$y=2+\sec\theta$

c. $\frac{(x+2)^2}{4}-\frac{(y-1)^2}{81}=1$ V(-2,1) $a=2$ $b=9$

$x=-2+2\sec\theta$
$y=1+9\tan\theta$

d. $y^2-5x^2+20x=50$

$y^2-5(x^2-4x+4)=50-20$

$y^2-5(x-2)^2=30$

$\frac{y^2}{30}-\frac{(x-2)^2}{4}=1$

V(2,0) $a=\sqrt{30}$ $b=\sqrt{6}$

$y=\sqrt{30}\sec\theta$
$x=2+\sqrt{6}\tan\theta$