Unit 12 Probability

15.1: Experimental and Theoretical Probability

The probability of an event is a number from 0 to 1 that describes how ____________________ it is that an event will occur. Probabilities closer to 1 are more likely to occur, and probabilities closer to 0 are less likely to occur.

Event: ____

Complement of an event: ____

Experimental Probability: ____

Outcome: ____

Sample Space: ____

Theoretical Probability: ____

Experimental Probability:

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{number of times the experiment is done}}$$

Theoretical Probability:

$$P\ (\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

The sum of the probability of an event and the probability of its complement is 1.

Calculating Experimental Probability

Example #1

A quality control inspector samples 500 LCD monitors and finds defects in three of them. What is the experimental probability that a monitor selected at random will have a defect?

Example #2

A park has 538 trees. You choose 40 at random and determine that 25 are maple trees. What is the experimental probability that a tree chosen at random is a maple tree? About how many trees in the park are likely to be maple trees?

Calculating Theoretical Probability

Example #1:

What is the probability of rolling numbers that add to 7 when rolling two standard number cubes (dice)?

Step 1: List or make a table for all possible results for the rolls of two number cubes.

	1	2	3	4	5	6
1	1,1	2,1	3,1	4,1	5,1	6,1
2	1,2	2,2	3,2	4,2	5,2	6,2
3	1,3	2,3	3,3	4,3	5,3	6,3
4	1,4	2,4	3,4	4,4	5,4	6,4
5	1,5	2,5	3,5	4,5	5,5	6,5
6	1,6	2,6	3,6	4,6	5,6	6,6

Step 2: Find the number of possible outcomes for the event that the sum of the two cubes is 7.

Step 3: Find the probability

$$P(\text{rolling a sum of 7}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

= ______

Example #2:
What is the probability of getting a sum of 13 when rolling two standard number cubes?

Example #3:
A jar contains 10 red marbles, 8 green marbles, 5 blue marbles, and 6 white marbles. What is the probability that a randomly selected marble is NOT green?

15.2 Geometric Probability

Vocabulary:

Geometric Probability: ____

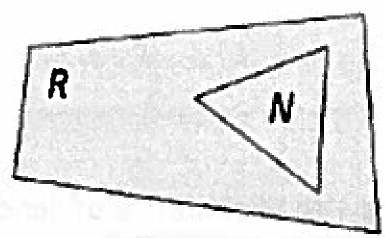
$$Probability = \frac{\#\ of\ favorable\ outcomes}{total\ possible\ outcomes}$$

take note

Key Concept Probability and Area

Point S in region R is chosen at random. The probability that S is in region N is the ratio of the area of region N to the area of region R.

$$P(S \text{ in region } N) = \frac{\text{area of region } N}{\text{area of region } R}$$



take note

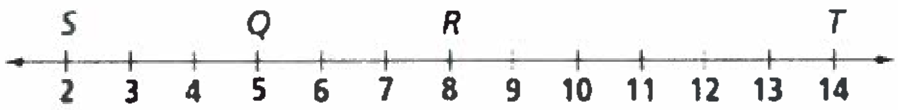
Key Concept Probability and Length

Point S on $\overline{AD}$ is chosen at random. The probability that S is on $\overline{BC}$ is the ratio of the length of $\overline{BC}$ to the length of $\overline{AD}$.

$$P(S \text{ on } \overline{BC}) = \frac{BC}{AD}$$

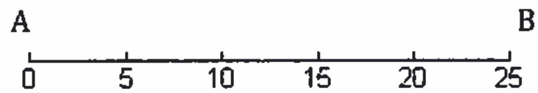
Example #1:

Point K on $\overline{ST}$ is chosen at random. What is the probability that K lies on $\overline{QR}$?



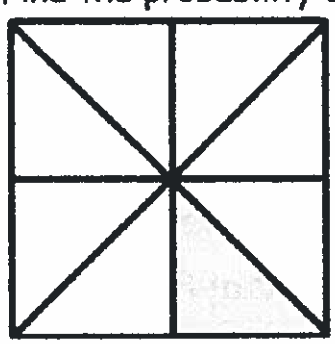
1. Matt's bus runs every 25 minutes. If he arrives at the bus stop at a random time, what is the probability that he will have to wait 10 minutes or more.

The 25 minute interval can be represented by $\overline{AB}$. Find P(waiting 10 or more)



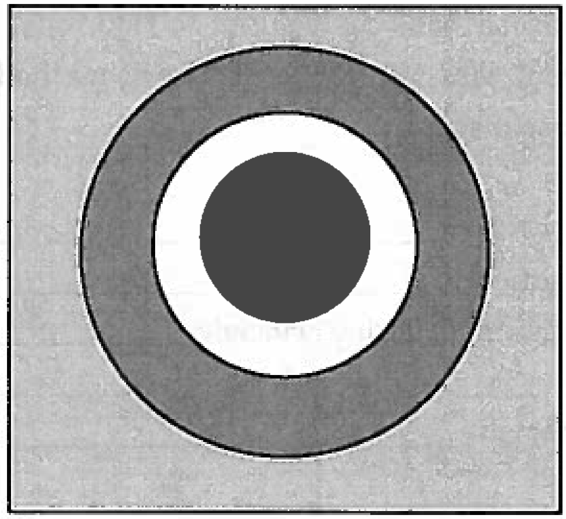
2. At the bus station, a bus is scheduled to arrive every 35 minutes. The bus waits for 5 minutes while passengers get off and on, and then departs for the next station. What is the probability that there is a bus waiting when a pedestrian arrives at the station at a random time?

3. Find the probability of landing in the shaded region.



This is a square.

4. Given Dart board 16 in x 16 in



Diameter red = 12 in
Diameter yellow = 9 in
Diameter blue = 6 in

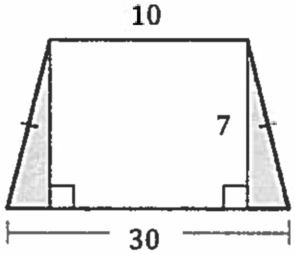
Find P(Blue)

Find P(yellow)

Find P(Red)

Find the probability that a randomly chosen point in the figure lies in the shaded region.

5.



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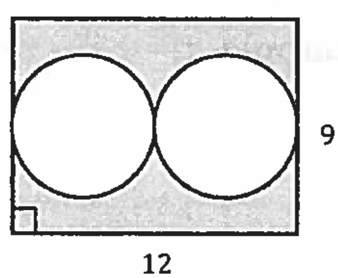


15.3 Permutations and Combinations

Counting methods:

Counting methods can be used to find the number of ways to choose objects from different sets. You can use counting methods to find the total number of outcomes in a sample space.

Permutation: ____

Combination: ____

Fundamental Counting Principle: ______________________________

____

N factorial: ____

take note

Key Concept Fundamental Counting Principle

The **Fundamental Counting Principle** says that if event M can occur in m ways and event N can occur in n ways, then event M followed by event N can occur in $m \cdot n$ ways.

Example

A menu with 4 entrees and 6 drinks has $4 \cdot 6 = 24$ possible lunch specials.

take note

Key Concept *n* Factorial

For any positive integer n, n factorial is $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$ and is written as $n!$. Zero factorial, or $0!$, is defined to be 1.

Example $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

Key Concept Permutation Notation

The number of permutations of n items of a set arranged r items at a time is

$${}_nP_r = \frac{n!}{(n-r)!} \text{ for } 0 \leq r \leq n.$$

Example

$${}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 8 \cdot 7 \cdot 6 = 336$$

Key Concept Combination Notation

The number of combinations of n items chosen r at a time is

$${}_nC_r = \frac{n!}{r!(n-r)!} \text{ for } 0 \leq r \leq n.$$

Example

$${}_9C_4 = \frac{9!}{4!(9-4)!} = \frac{9!}{4!5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(4 \cdot 3 \cdot 2 \cdot 1)(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 126$$

Example #1:

A deli offers a lunch special if you choose one from each of the following types of sandwiches, side items, and drink choices. How many different lunch specials are possible?

There are 6 possible sandwiches, 4 different side items, and 5 different drink choices. Use the fundamental counting principle!

Example #2:

You can download 8 songs on your music player. If you play the songs using the random shuffle option, how many different ways can the sequence of songs be played? (Hint: Use the Fundamental Counting Principle).

Example #3:

In how many ways can you arrange 12 books on a shelf?

Using the Permutation Formula:

Example #4:

The Student Council is electing a president, a vice president, and a treasurer. How many different ways can the officers be chosen from the 10 members?

Method 1: Use the formula for permutations. There are 10 members, arranged 3 at a time. So n=10 and r=3.

$$ {}_{10}P_3 = \frac{10!}{(10-3)!} $$

Method 2: Use a graphing calculator.

Press 1 0 math ◁ 2 3 enter

Example #5:

Twelve swimmers compete in a race. In how many possible ways can the swimmers finish first, second, and third?

Using the Combination Formula:

Example #6:

Suppose your school requires you to choose 4 books to read on summer vacation from a reading list of 12 books. How many different ways to choose the books are possible?

Method 1: Use the formula for finding combinations

There are 12 books, chosen 4 at a time.

$$ {}_{12}C_4 = \frac{12!}{4!(12-4)!} $$

Method 2: Use a graphing calculator.

Press [1] [2] [math] ◁ [3] [4] [enter]

Example #7:

A service club has 8 freshmen. Five of the freshman are to be on the clean-up crew for the town's annual picnic. How many different ways are there to choose the 5 member clean-up crew?

How do you know when to choose the permutation formula or the combination formula? ______________________________

Practice on the following problems:

- A college student is choosing 3 classes to take during the first, second, and third semesters from the 5 elective classes offered in his major. How many possible ways can the student choose the three classes?
- A jury of 12 people is chosen from a pool of 35 potential jurors. How many different juries can be chosen?
- A yogurt shop allows you to choose any 3 of the 10 possible mix-ins for a smoothie. How many different smoothies are possible?

15.4: Compound Probability

Compound event: ______________________________

Dependent event: ______________________________

Independent event: ______________________________

Mutually exclusive events: ______________________________

____

Overlapping events: ______________________________

A compound event is made up of two or more events. The outcomes of ______________ events do not affect each other. If the outcome of an event affects the outcome of another event, they are ______________ events.

Mutually exclusive events cannot occur at the same time.

take note

Key Concept Probability of Independent Events

If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

take note

Key Concept Probability of Mutually Exclusive Events

If A and B are mutually exclusive events, then $P(A \text{ and } B) = 0$, and $P(A \text{ or } B) = P(A) + P(B)$.

take note

Key Concept Probability of Overlapping Events

If A and B are overlapping events, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Are the following events independent or dependent events?

- Choose a number tile from 12 tiles. Then spin a spinner.
- Pick one card from a set of sequentially numbered cards. Then, without replacing the card, pick another card.
- You role a dice, then flip a coin.

Example #1:

A desk drawer contains 5 red pens, 6 blue pens, 3 black pens, 24 silver paper clips, and 16 white paper clips. If you select a pen and a paper clip from the drawer without looking, what is the probability that you select a blue pen and a white paper clip?

Step 1: Let A= selecting a blue pen. Find the probability of A.

P(A) =

Step 2: Let B= selecting a white paper clip. Find the probability of B.

P(B)=

Step 3: Find P(A and B).

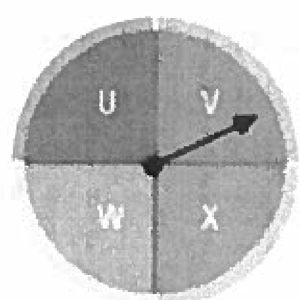
Use the formula for the probability of independent events.

P(A and B) = P(A) x P(B)

Therefore the probability that you select a blue pen and a white paper clip is about ___________ percent.

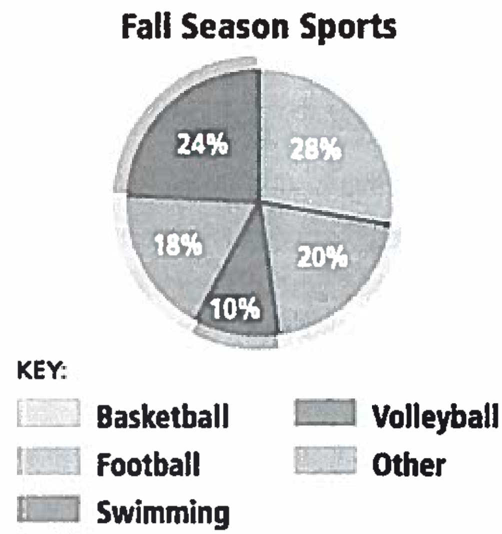
Example #2:

You roll a standard number cube and spin the spinner at the right. What is the probability that you roll a number less than 3 and the spinner lands on a vowel?



Example #3:

Student athletes at a local high school may participate in only one sport each season. What is the probability that a randomly selected student athlete plays volleyball OR is on the swim team?



Because athletes participate in only one sport each season, the events are mutually exclusive.

Use the formula P(A or B) = P(A) + P(B)
P(volleyball or swim)= P(volleyball) + P(swim)

Example #3:

Student athletes at a local high school may participate in only one sport each season. In the spring season, 15% of the athletes play baseball and 23% are on the track team. What is the probability of an athlete either playing baseball or being on the track team?

Finding the Probabilities of Overlapping Events:

Example #4:

What is the probability of rolling either an even number OR a multiple of 3 when rolling a standard number cube?

Step 1: Find the P(even)
Step 2: Find the P(multiple of 3)
Step 3: Find the P(even AND multiple of 3)
Step 4: Plug in to P(even) + P(multiple of 3) – P(even and multiple of 3)

Example #5:
What is the probability of rolling either an odd number or a number is less than 4 when rolling a standard number cube?

15.5 Conditional Probability and Frequency Tables

take note

Key Concept Conditional Probability

The probability that an event B will occur, given that another event A has already occurred, is called a **conditional probability** and is written $P(B|A)$. You read $P(B|A)$ as "the probability of event B, given event A."

Example You randomly select two marbles, one at a time, from a bag containing 3 red marbles and 5 green marbles. If your first marble is red, then 2 of the remaining 7 marbles are red, so $P(\text{2nd marble is red} \mid \text{1st marble is red}) = \frac{2}{7}$.

Frequency Tables

A frequency table is a data display that shows how often an item appears in a particular category.

- The relative frequency of an item is the ratio number of times the item occurs to the total number of items in the sample space.
- A probability distribution can be shown in a frequency table.

Satisfaction Level	Very Satisfied	Somewhat Satisfied	Somewhat Unsatisfied	Very Unsatisfied
Frequency	8	4	3	1
Probability	0.5	0.25	0.1875	0.0625

Two-Way Frequency Tables

A two-way frequency table, or contingency table, is a data display that shows the frequencies of data in two different categories.

	Plan to Attend College	Do not Plan to Attend College	Totals
Juniors	29	3	32
Seniors	33	5	38
Totals	62	8	70

Example #1:

The table shows data about student involvement in extracurricular activities at a local high school. What is the probability that a randomly chosen student is a female who is not involved in extracurricular activities?

Extracurricular Activities

	Involved in Activities	Not Involved in Activities	Totals
Male	112	145	257
Female	139	120	259
Totals	251	265	516

To find probability, calculate relative frequency.

Relative frequency= $\dfrac{\text{females not involved}}{\text{total number of students}}$ =

Example #2:

The two-way frequency table below shows the number of male and female students by grade level on the prom committee. What is the probability that a member of the prom committee is a male who is a junior?

	Male	Female	Totals
Juniors	3	4	7
Seniors	3	2	5
Totals	6	6	12

Example #3:

Respondents of a poll were asked whether they were for, against, or had no opinion about a bill before the state legislature that would increase the minimum wage. What is the probability that a randomly selected person is over 60 years old, given that the person had no opinion on the state bill?

Age Group	For	Against	No Opinion	Totals
18–29	310	50	20	380
30–45	200	30	10	240
46–60	120	20	30	170
Over 60	150	20	40	210
Totals	780	120	100	1000

Example #4:

A company has 150 representatives. Two months after a sales seminar, the company vice-president made the table of relative frequencies based on sales results. What is the probability that someone who attended the seminar had an increase in sales?

	Attended Seminar	Did not Attend Seminar	Totals
Increased Sales	0.48	0.02	0.5
No Increase in Sales	0.32	0.18	0.5
Totals	0.8	0.2	1

Find P(increased sales | sales seminar)

Example #5

Use the two-way frequency table to find P(democrat and supports the issue) and P(democrat | supports the issue).

	Supports the Issue	Does Not Support the Issue	Totals
Democrat	24	36	60
Republican	27	33	60
Totals	51	69	120

Name ______________________________ Class ____________ Date ____________

15-1 Practice

Form K

Experimental and Theoretical Probability

You roll a standard number cube 8 times. The results are shown below.

5, 1, 2, 4, 6, 3, 5, 5

Find the experimental probability of each outcome.

1. *P*(rolling a 5)

   Number of 5s rolled: __________

   Total number of rolls: __________

   Experimental probability: $\frac{\text{Number of 5s}}{\text{Number of rolls}} =$ _____

2. *P*(rolling a 6)

   Experimental probability: $\frac{\text{Number of 6s}}{\text{Number of rolls}} =$ _____

3. *P*(rolling an even number)

4. What is the experimental probability of rolling a multiple of 3 on a standard number cube? For 60 rolls of the number cube, predict the number of rolls that will result in a multiple of 3.

Find the theoretical probability of each outcome.

5. *P*(rolling a 5)

6. *P*(rolling a 6)

7. *P*(rolling an even number)

8. *P*(rolling a multiple of 3)

A bag contains 1 red marble, 3 green marbles, 1 blue marble, and 1 yellow marble. Suppose one marble is picked at random. Find each probability.

9. *P*(blue)

10. *P*(not green)

11. *P*(not red)

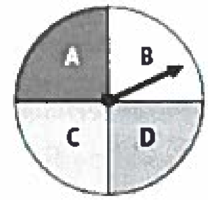
12. *P*(not yellow)

Name ______________________ Class ____________ Date ____________

15-1 Practice (continued) Form K

Experimental and Theoretical Probability

13. A spinner has 4 equal sections. After 12 spins, the spinner landed on section A 4 times, section B 5 times, section C 2 times, and section D 1 time.



a. What is the experimental probability of the spinner stopping on section A?

b. What is the theoretical probability of the spinner stopping on section A?

14. **Reasoning** If the probability of an event occurring is $\frac{1}{4}$, what is the probability of its complement?

Two standard number cubes are rolled. Find each probability.

15. P(a sum equal to 3)

16. P(a sum not equal to 3)

17. P(a sum equal to 12)

18. **Writing** An event has a probability of 1. What does this tell you about the event? Explain.

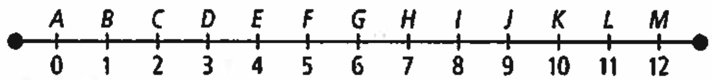
19. **Error Analysis** You and a friend flip a coin 10 times. The coin lands on heads 7 times. Your friend says that the theoretical probability of getting heads is $\frac{7}{10}$. What error did your friend make? What is the correct value for theoretical probability? Explain.

Name________________________________ Class____________ Date ____________

15-2 Practice

Form K

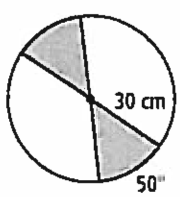
A point on $\overline{AM}$ is chosen at random. Find the probability that the point lies on the given segment.



1. $\overline{DJ}$
2. $\overline{JL}$
3. $\overline{BE}$
4. $\overline{CK}$
5. $\overline{AJ}$
6. $\overline{BL}$

7. A fitness club set up an express exercise circuit. To warm up, a person works out on weight machines for 90 s. Next the person jogs in place for 60 s, and then takes 30 s to do aerobics. After this, the cycle repeats. If you enter the express exercise circuit at a random time, what is the probability that a friend of yours is jogging in place? What is the probability that your friend will be on the weight machines?

A point in the figure is chosen at random. Find the probability that the point lies in the shaded region.

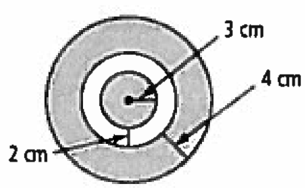
8. 

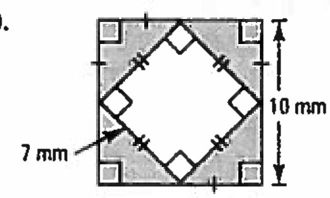
To start, find the area of the two shaded sectors.

Then find the total area.

Area of two sectors $= 2 \cdot \frac{50}{360} \cdot \pi \cdot \square^2$

Total area $= \pi \cdot \square^2$

9. 

10. 

11. A Sunday night sports show is on from 10:00 P.M. to 10:30 P.M. You want to find out if your favorite team won this weekend, but forgot that the show was on. You turn it on at 10:14 P.M. The score will be announced at one random time during the show. What is the probability that you haven't missed the report about your favorite team?

Name________________________ Class ____________ Date____________

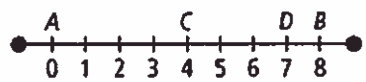
15-2 Practice (continued)

Form K

A point between *A* and *B* on each number line is chosen at random. What is the probability that the point is between *C* and *D*?

12.

13.



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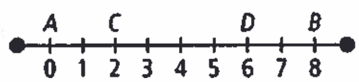

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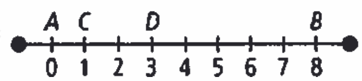
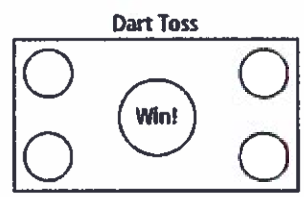


16. In the fundraiser game at the right, players toss darts at a board to try to get them into one of the holes. The diameter of the center hole is 8 in. The diameter of each of the four corner holes is 5 in. The board is a 20-in.-by-30-in. rectangle. Find the probability that a tossed dart will go through the indicated hole.



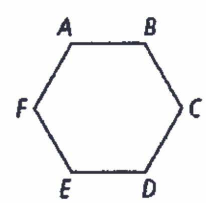
a. center hole

b. top right or left corner

c. any corner

d. lower left corner

17. Reasoning Suppose a point on the perimeter of the regular hexagon at the right is chosen at random. What is the probability that the point lies on $\overline{AB}$ or $\overline{BC}$? Explain.



18. At the space museum, a movie starts every 15 min. There are 5 min between shows. If you enter the theater at a random time, what is the probability that you will have to wait more than 2 min. for the next movie to start?

Name ______________________________ Class ______________ Date ______________

15-3 Practice

Form G

Permutations and Combinations

1. A band sells t-shirts in 3 sizes and 2 different colors. How many different t-shirts are there to choose from?

2. Each player on the baseball team can order a baseball bat using the table to the right. How many choices does each player have?

Finish	Length	Wood Type
Natural	32"	Ash
Black	33"	Maple
	34"	

3. In how many different orders can 5 runners finish a race?

4. Evaluate 7!.

5. What is the value of $\frac{25!}{24!}$?

6. How many possible combinations of 3 items from a group of 5 are possible?

7. Evaluate ${}_6P_3$.

8. A basketball coach will choose 5 players from a group of 8 players to start the next game. How many different groups of starting players are possible?

9. What is the value of ${}_nC_r$ when $n = 7$ and $r = 4$?

10. What is the probability of randomly choosing a penny and a nickel from a cup of coins that contains a penny, a nickel, a dime, and a quarter?

11. Three playing cards are randomly chosen from a set numbered from 1 to 7. What is the probability that the chosen cards are numbered 1, 2 and 3?

Name ______________________________ Class ______________ Date ______________

15-3 Practice (continued)

Form G

Permutations and Combinations

12. **Recreation** When renting a bike from a local bike shop, you can choose from the types, sizes, and colors in the table shown below?

Type	Size	Color
Mountain	Small	Green
Cruising	Medium	Red
Road	Large	Blue

How many different choices do you have?

13. **Open-Ended** Use an example to explain why you can use $n!$ to find the number of possible orders for n objects.

14. **Reasoning** A hiker has 2 pairs of hiking shoes, 3 different shirts, and 2 different pairs of shorts to choose from. How does the number of combinations of shoes, shirts, and shorts change as the hiker adds shirts to his collection? Explain.

15. **Business** For each weekly meeting of a group of business leaders, members take turns being the note-taker, the facilitator, and the speaker. In how many different ways can these positions be chosen from the 9 members?

16. **Writing** Explain how the Fundamental Counting Principle is related to a tree diagram.

17. A game at the fair involves ping-pong balls numbered 1 to 18. You can win a prize if you correctly choose the 5 numbers that are randomly drawn. What are your chances of winning?