Triangle Inequality Theorem and Triangles in Spherical Geometry (5.7 & 3.9)

To be or not to be???

Use constructions to verify if a triangle can be formed with the given side lengths or not.

1. 3 cm, 6 cm and 7 cm

2. 3 cm, 5 cm and 8 cm

3. 3 cm, 4 cm, 8 cm

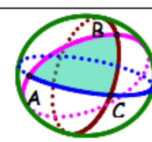
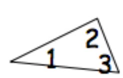
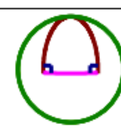
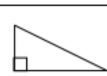
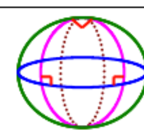
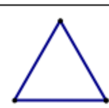
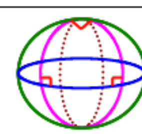
4. 4 cm, 5 cm, 7 cm

What must be true about the lengths of the sides in order for a triangle to be formed?

____

____

Spherical Geometry VS Euclidean Geometry:

Euclidean Geometry		Spherical Geometry	
3 non-collinear points form a triangle		3 non-collinear points form a triangle	
Triangle sum is 180^0.		Triangle sum is greater than 180 and less than 540	
A triangle can have one obtuse angle (one right angle)		A triangle can have more than one obtuse angle (one right angle)	
Each angle measure of an equiangular triangle is 60.		Each angle measure of an equiangular triangle can vary.	

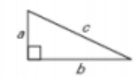
Pythagorean Theorem

Pythagorean Triple ____

____

Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$c^2 = a^2 + b^2$

You may find it helpful to memorize the basic Pythagorean triples, shown in **bold**, for standardized tests.

COMMON PYTHAGOREAN TRIPLES AND SOME OF THEIR MULTIPLES

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
30, 40, 50	50, 120, 130	80, 150, 170	70, 240, 250
$3x$, $4x$, $5x$	$5x$, $12x$, $13x$	$8x$, $15x$, $17x$	$7x$, $24x$, $25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold face triple by the same factor.

Samples:

1. The hypotenuse of a right triangle has length 12. One leg has length 6. What is the length of the other leg?

2. A triangle has side lengths of 16, 48, and 50. Is the triangle a right triangle? Explain.

3. Find a third whole number such that the three numbers form a pythagorean triple:

20, 21 14,48

10.1 Pythagorean Theorem and its converse

Goal

- Use the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle.

Converse of the Pythagorean Theorem:

If c^2 ____ $a^2 + b^2$, then $\triangle ABC$ is a ______ triangle.

Corollary:

If c^2 ____ $a^2 + b^2$, then $\triangle ABC$ is a ______ triangle.

Corollary:

If c^2 ____ $a^2 + b^2$, then $\triangle ABC$ is a ______ triangle.

Applying Converse of the Pythagorean Theorem

Tell whether the triangle formed with the given sides is acute, obtuse, or right. If a triangle can't be formed write No triangle.

1. 5, 12, 13
2. $\sqrt{8}$ 4, 6
3. 20, 21, 28
4. 15, 36, 39
5. $\sqrt{13}$, 10, 12
6. 14, 48, 50
7. 10, 12, 30
8. 16, 30, 34
9. 18, 34, 45

Geometric Mean:

What is the geometric mean of 6 and 15?

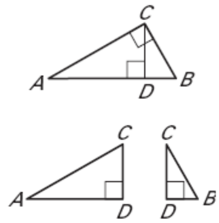
Summary

9-4 Use Similar Right Triangles

Goal • Use properties of the altitude of a right triangle.

THEOREM 7.5

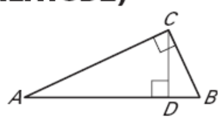
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.

THEOREM 7.6: GEOMETRIC MEAN (ALTITUDE) THEOREM

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

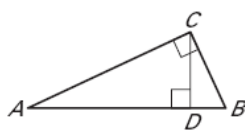


The length of the altitude is the geometric mean of the lengths of the two segments.

$$\frac{BD}{CD} = \frac{CD}{AD}$$

THEOREM 7.7: GEOMETRIC MEAN (LEG) THEOREM

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

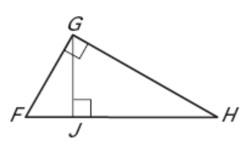


The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

$$\frac{AB}{CB} = \frac{CB}{DB}$$ and

$$\frac{AB}{AC} = \frac{AC}{AD}$$

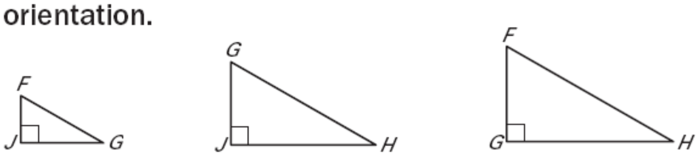
Identify the similar triangles in the diagram.



Solution

Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.

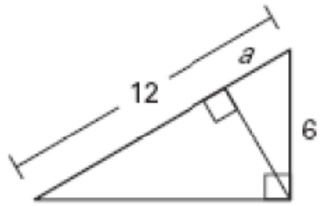
1.



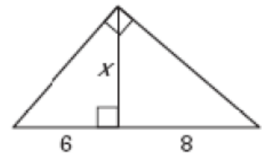
Summary $\triangle$_____ $\sim$ $\triangle$_____ $\sim$ $\triangle$_____

9.4 Use Similar Right Triangles

2.



3.



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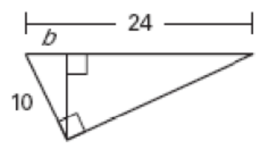

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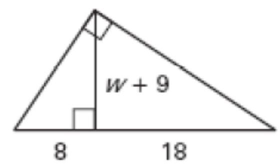
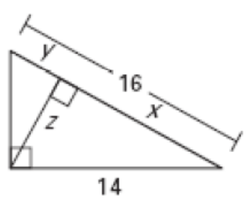


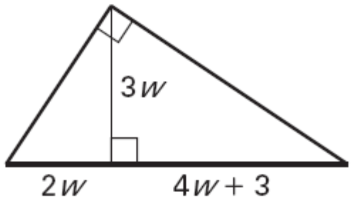
6. Solve for x, y and Z. Round to the nearest hundredth.



Summary

9.4 Use Similar Right Triangles

7.



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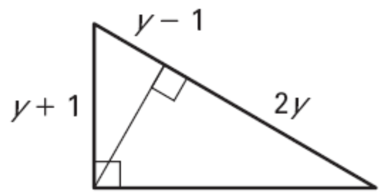
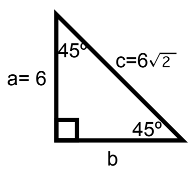
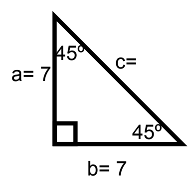


Use the pythagorean theorem to solve for the missing sides in simplest radical form.

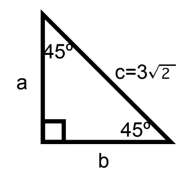
a=___ b=____ c=____



a=___ b=____ c=____



a=___ b=____ c=____

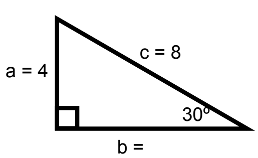


Do you see a pattern for 45-45-90 right triangle?

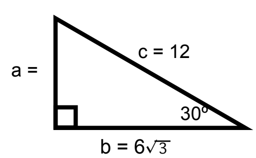
______*leg = hypotenuse

Use the pythagorean theorem to solve for the missing sides in simplest radical form.

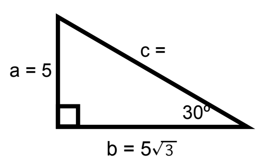
a=___ b=____ c=____



a=___ b=____ c=____



a=___ b=____ c=____



Do you see a pattern for 30-60-90 triangles?

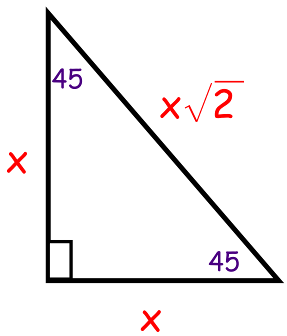
_____* short leg = long leg   _____* short leg = hypotenuse

Summary

Practice: 45-45-90 triangles

45 - 45 - 90

$x \quad x \quad x\sqrt{2}$



x	x	$x\sqrt{2}$
45	45	90
5		
	8	
		$10\sqrt{2}$
$6\sqrt{3}$		
		8
		14
		15

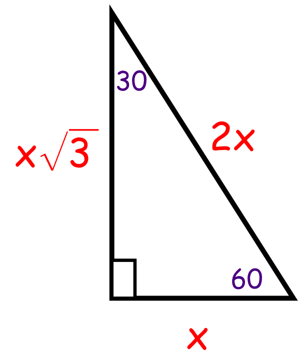
x	x	$x\sqrt{2}$
45	45	90
		$7\sqrt{10}$
		17
	3	
$5\sqrt{6}$		
	$3\sqrt{2}$	
		30
		100
1		
		$12\sqrt{2}$

x	x	$x\sqrt{2}$
45	45	90
7		
9		
		$6\sqrt{2}$
		12
	9	
	$8\sqrt{2}$	
$4\sqrt{2}$		
	$4\sqrt{5}$	
		18

Practice: 30-60-90 Triangles

30 - 60 - 90

x $x\sqrt{3}$ $2x$



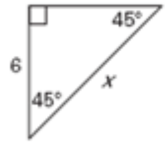
30	60	90
x	$x\sqrt{3}$	$2x$
6		
	$5\sqrt{3}$	
		8
$2\sqrt{3}$		
	$7\sqrt{3}$	
		18
	15	
	18	

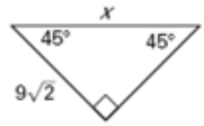
30	60	90
x	$x\sqrt{3}$	$2x$
$2\sqrt{3}$		
	$8\sqrt{3}$	
		10
$8\sqrt{3}$		
	$8\sqrt{3}$	
		15
	14	
	10	

10.2 Special Right Triangles

Using 45, 45, 90 Triangle Rule

Find the value of x. Write your answer in simplest radical form.

1.


2.


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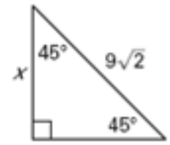

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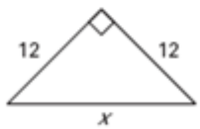
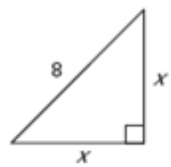


5.


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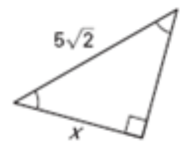
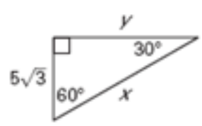


Using 30, 60, 90 Triangle Rule

Find the value of each variable. Write your answers in simplest radical form.

7.


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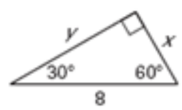
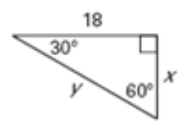
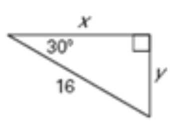
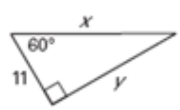
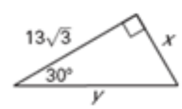


9.


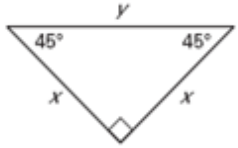
10.


11.


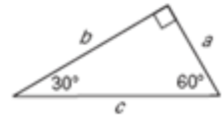
12.


Applying Special Triangle Rules

Complete the table.

13.


x	5		$\sqrt{2}$	9	
y		$4\sqrt{2}$			24

14.


a	9			11	
b		9	$5\sqrt{3}$		
c					16

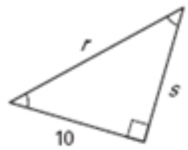
Summary

10.2 Special Right Triangles

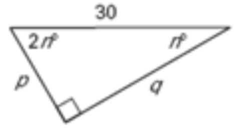
Using Special Triangles

Find the value of each variable. Write your answers in simplest radical form.

15.



16.



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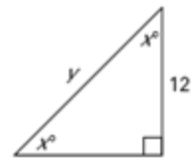
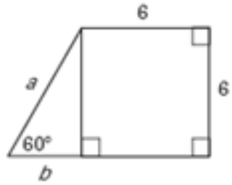
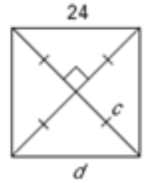


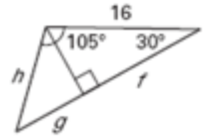
18.



19.



20.



The side lengths of a triangle are given. Determine whether it is a *45°-45°-90° triangle*, a *30°-60°-90° triangle*, or *neither*.

21. $5, 10, 5\sqrt{3}$

22. $7, 7, 7\sqrt{3}$

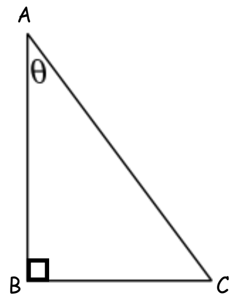
23. $6, 6, 6\sqrt{2}$

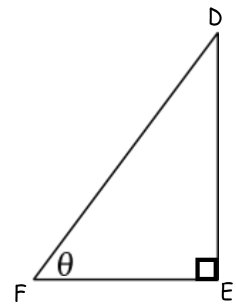
Summary

10-3 Trigonometry

Trigonometric Ratio -

Label the parts of triangles



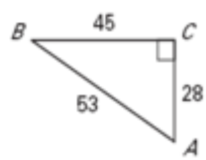


Sine Cosine Tangent

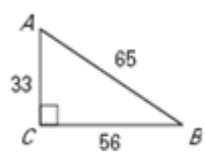
S_ _ C _ _ T _ _

Find tan *A* and tan *B*.

1.

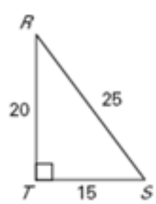


2.

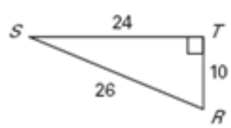


Find sin *R* and sin *S*.

3.

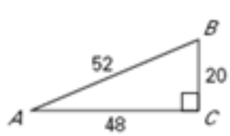


4.

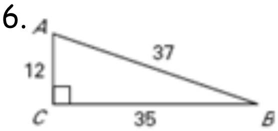


Find cos *A* and cos *B*.

5.



6.



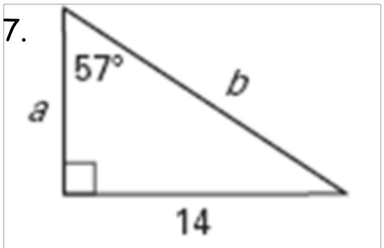
Summary

Make sure your calculator is in degree mode!!!

Examples

Label each leg & write each ratio to solve.

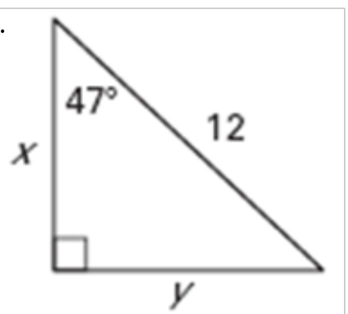
7.



Find a:

Find b:

8.



Find x:

Find y:

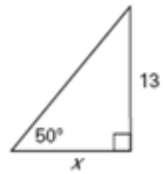
Summary

10-3 Trigonometry

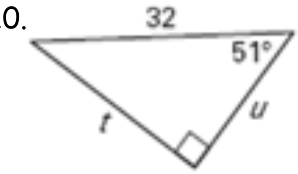
Find missing side

Find the value of *x* to the nearest tenth.

9.



10.



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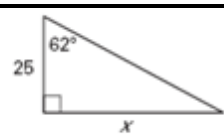

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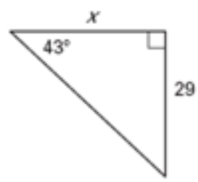

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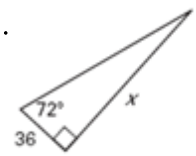
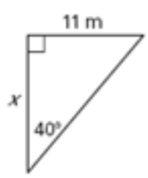


Using trig ratio and finding area of a triangle.

Find the area of the triangle. Round your answer to the nearest hundredth.

14.



15.

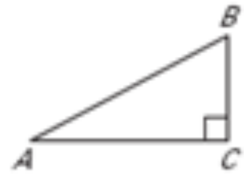
16.

Summary

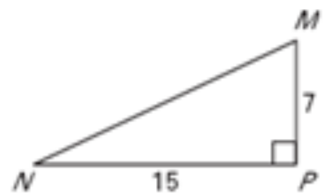
10-3 Trigonometry

Goal • Use inverse tangent, sine, and cosine ratios.

Solve a right triangle –



Use the diagram to find the indicated measurement. Round your answer to the nearest tenth.



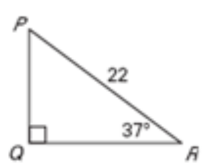
1. MN

2. $m\angle M$

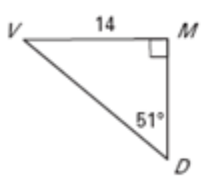
3. $m\angle N$

Solve the right triangle. Round your answers to the nearest tenth.

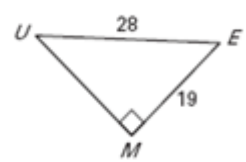
4.



5.



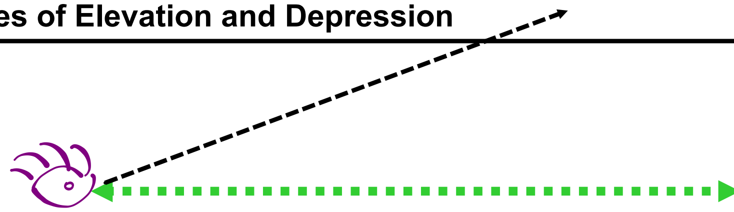
6.



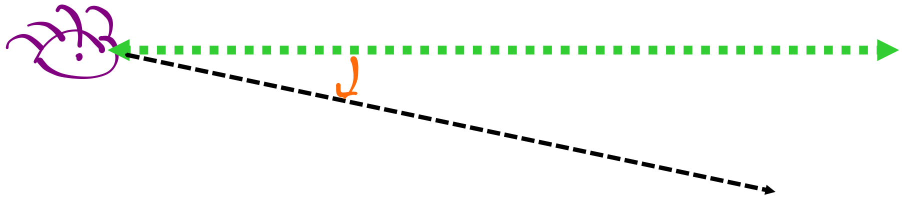
Summary

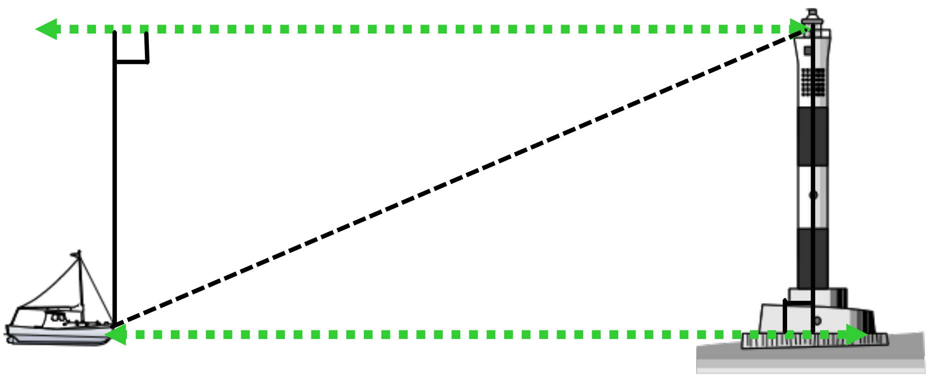
10-4 Angles of Elevation and Depression

Angle of Elevation –



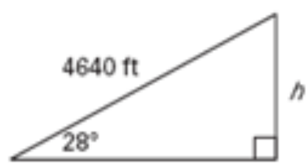
Angle of Depression –





What do you notice about the angle of depression and angle of elevation?

1. Ski Lift A chair lift on a ski slope has an angle of elevation of 28° and covers a total distance of 4640 feet. To the nearest foot, what is the vertical height h covered by the chair lift?

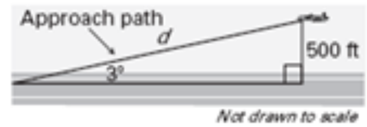


Summary

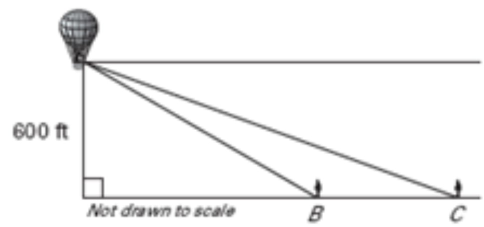
Applying Angle of Elevation & Depression	

2.

Airplane Landing You are preparing to land an airplane. You are on a straight line approach path that forms a 3° vertical angle with the runway. What is the distance *d* along this approach path to your touchdown point when you are 500 feet above the ground? Round your answer to the nearest foot.



Hot Air Balloon: You are in a hot air balloon that is 600 feet above the ground where you can see two people.



3. If the angle of depression from your line of sight to the person at B is 30°, how far is the person from the point on the ground below the hot air balloon?

4. If the angle of depression from your line of sight to the person at C is 20°, how far is the person from the point on the ground below the hot air balloon?

5. How far apart are the two people?

Summary