

Find the intercepts, asymptotes, domain, range, holes, end behavior and behavior at each point of discontinuity of the rational equations (when applicable) and then graph.

1. $f(x) = \frac{6}{x-3}$

x-int none y-int (0, -2)

VA x=3 HA/OA y=0

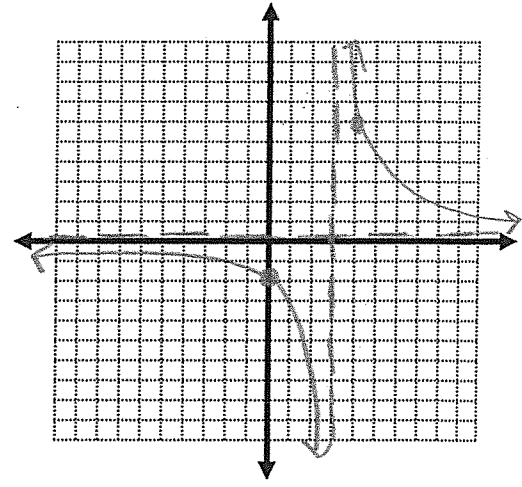
Domain $(-\infty, 3) \cup (3, \infty)$ Range $(-\infty, 0) \cup (0, \infty)$

Hole none End Behavior _____

as $x \rightarrow \infty, y \rightarrow 0$ / as $x \rightarrow -\infty, y \rightarrow 0$

Behavior at each point of discontinuity _____

as $x \rightarrow 3^-, y \rightarrow -\infty$ / as $x \rightarrow 3^+, y \rightarrow +\infty$



2. $f(x) = \frac{x+3}{x-5}$

x-int (-3, 0) y-int (0, -3/5)

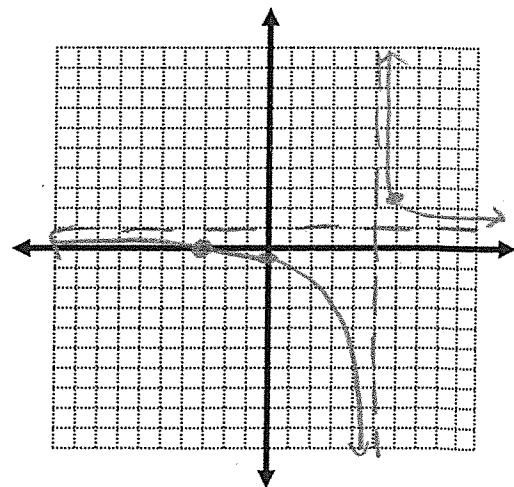
VA x=5 HA/OA y=1

Domain $(-\infty, 5) \cup (5, \infty)$ Range $(-\infty, 1) \cup (1, \infty)$

Hole none End Behavior as $x \rightarrow \infty, y \rightarrow 1$ / as $x \rightarrow -\infty, y \rightarrow 1$

Behavior at each point of discontinuity _____

as $x \rightarrow 5^-, y \rightarrow -\infty$ / as $x \rightarrow 5^+, y \rightarrow \infty$



3. $f(x) = \frac{2x-1}{4x+3}$

x-int $(\frac{1}{2}, 0)$ y-int $(0, -\frac{1}{3})$

VA $x = -\frac{3}{4}$ HA/OA $y = \frac{1}{2}$

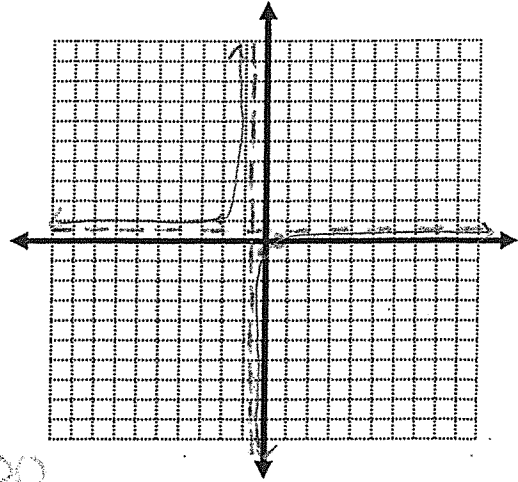
Domain $(-\infty, -\frac{3}{4}) \cup (-\frac{3}{4}, \infty)$ Range $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

Hole none End Behavior _____

as $x \rightarrow \infty, y \rightarrow \frac{1}{2}$ / as $x \rightarrow -\infty, y \rightarrow \frac{1}{2}$

Behavior at each point of discontinuity _____

as $x \rightarrow -\frac{3}{4}^-, y \rightarrow \infty$ | as $x \rightarrow -\frac{3}{4}^+, y \rightarrow -\infty$



4. $f(x) = \frac{x}{x^2-9}$

x-int $(0, 0)$ y-int $(0, 0)$

VA $x = \pm 3$ HA/OA $y = 0$

Domain $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$ Range $(-\infty, \infty)$

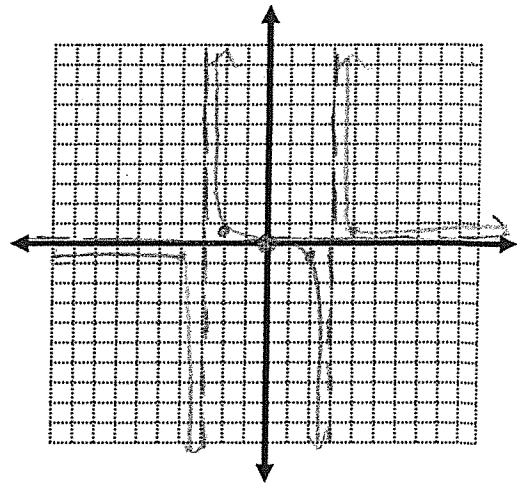
Hole none End Behavior _____

as $x \rightarrow \infty, y \rightarrow 0$ / as $x \rightarrow -\infty, y \rightarrow 0$

Behavior at each point of discontinuity _____

as $x \rightarrow -3^-, y \rightarrow -\infty$ / as $x \rightarrow -3^+, y \rightarrow \infty$

as $x \rightarrow 3^-, y \rightarrow -\infty$ / as $x \rightarrow 3^+, y \rightarrow \infty$



5. $f(x) = \frac{x}{x^2-x-2} = \frac{x}{(x-2)(x+1)}$

x-int $(0, 0)$ y-int $(0, 0)$

VA $x = 2, x = -1$ HA/OA $y = 0$

Domain $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$ Range $(-\infty, \infty)$

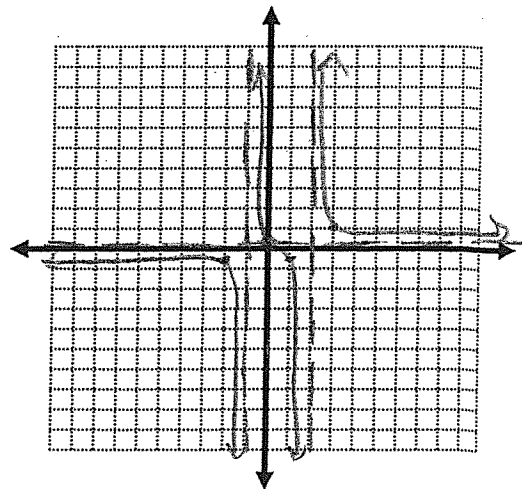
Hole none End Behavior _____

as $x \rightarrow \infty, y \rightarrow 0$ / as $x \rightarrow -\infty, y \rightarrow 0$

Behavior at each point of discontinuity _____

as $x \rightarrow -1^-, y \rightarrow -\infty$ / as $x \rightarrow -1^+, y \rightarrow \infty$

as $x \rightarrow 2^-, y \rightarrow -\infty$ / as $x \rightarrow 2^+, y \rightarrow \infty$



$$6. f(x) = \frac{x^2 - 5x + 4}{x^2 - 4} = \frac{(x-4)(x-1)}{(x+2)(x-2)}$$

x-int (4,0)(1,0) y-int (0,-1)

VA $x = \pm 2$ HA/OA $y = 1$

Domain $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$ Range $(-\infty, \infty)$

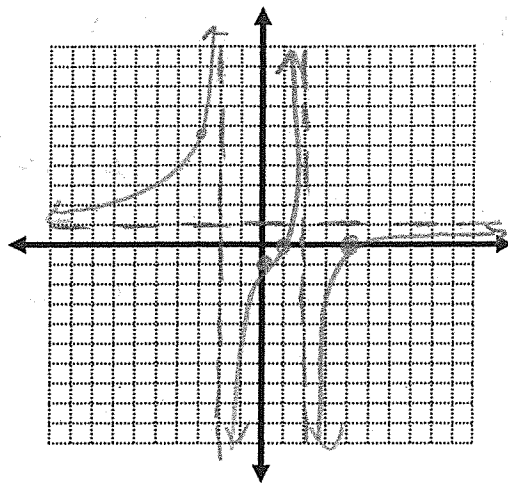
Hole none End Behavior _____

$as x \rightarrow \infty, y \rightarrow 1$ / $as x \rightarrow -\infty, y \rightarrow 1$

Behavior at each point of discontinuity _____

$as x \rightarrow -2^-, y \rightarrow \infty$ / $as x \rightarrow -2^+, y \rightarrow -\infty$

$as x \rightarrow 2^-, y \rightarrow \infty$ / $as x \rightarrow 2^+, y \rightarrow -\infty$



$$7. f(x) = \frac{x^2 - 2x - 8}{x^2 - 9} = \frac{(x-4)(x+2)}{(x+3)(x-3)}$$

x-int (4,0)(-2,0) y-int $(0, 8/9)$

VA $x = \pm 3$ HA/OA $y = 1$

Domain $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$ Range $(-\infty, \infty)$

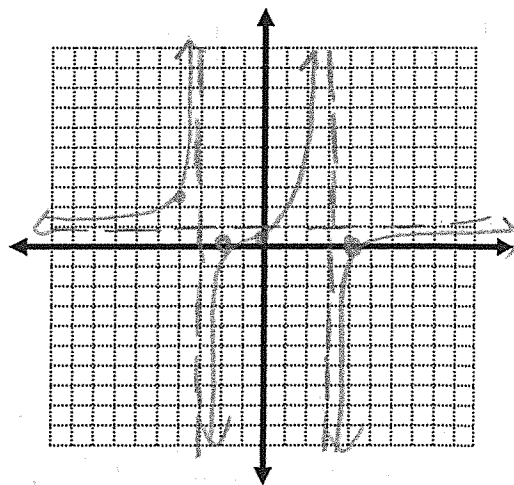
Hole none End Behavior _____

$as x \rightarrow \infty, y \rightarrow 1$ / $as x \rightarrow -\infty, y \rightarrow 1$

Behavior at each point of discontinuity _____

$as x \rightarrow -3^-, y \rightarrow \infty$ / $as x \rightarrow -3^+, y \rightarrow -\infty$

$as x \rightarrow 3^-, y \rightarrow \infty$ / $as x \rightarrow 3^+, y \rightarrow -\infty$



$$8. f(x) = \frac{4x-1}{-2x+3}$$

x-int (1/4, 0) y-int $(0, -1/3)$

VA $x = 3/2$ HA/OA $y = -2$

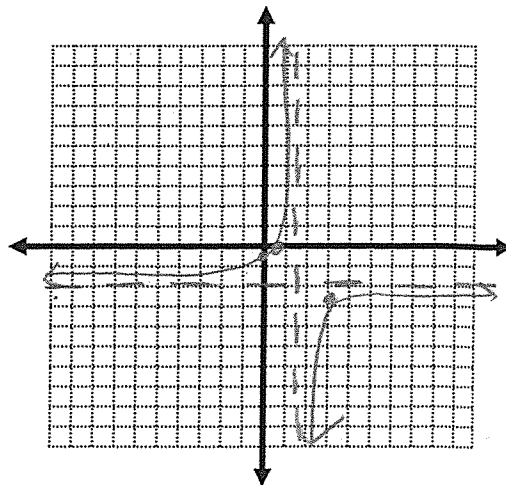
Domain $(-\infty, 3/2) \cup (3/2, \infty)$ Range $(-\infty, -2) \cup (-2, \infty)$

Hole none End Behavior _____

$as x \rightarrow \infty, y \rightarrow -2$ / $as x \rightarrow -\infty, y \rightarrow -2$

Behavior at each point of discontinuity _____

$as x \rightarrow 3/2^-, y \rightarrow \infty$ / $as x \rightarrow 3/2^+, y \rightarrow -\infty$



$$9. f(x) = \frac{x+2}{x^2-4} = \frac{(x+2)}{(x+2)(x-2)} = \frac{1}{x-2}$$

x-int none y-int (0, -1/2)

VA x=2 HA/OA y=0

Domain (-∞, -2) ∪ (-2, 2) ∪ (2, ∞) Range (-∞, -1/4) ∪ (-1/4, 0) ∪ (0, ∞)

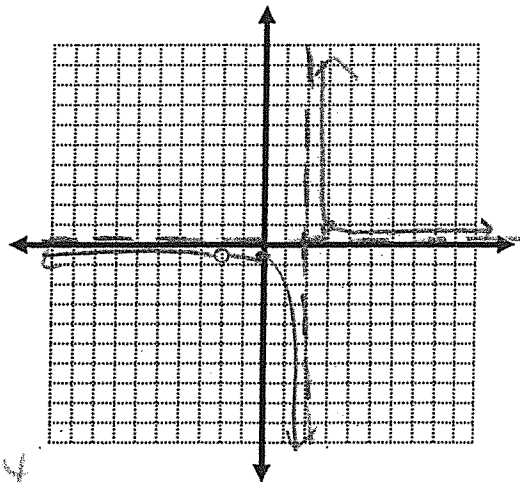
Hole (-2, -1/4) End Behavior _____

as x → ∞, y → 0 / as x → -∞, y → 0

Behavior at each point of discontinuity _____

as x → -2⁻, y → -1/4 / as x → -2⁺, y → -1/4

as x → 2⁻, y → -∞ / as x → 2⁺, y → ∞



$$10. f(x) = \frac{x}{3x^2-x} = \frac{x}{x(3x-1)} = \frac{1}{3x-1}$$

x-int none y-int none

VA x=1/3 HA/OA y=0

Domain (-∞, 0) ∪ (0, 1/3) ∪ (1/3, ∞) Range (-∞, -1) ∪ (-1, 0) ∪ (0, ∞)

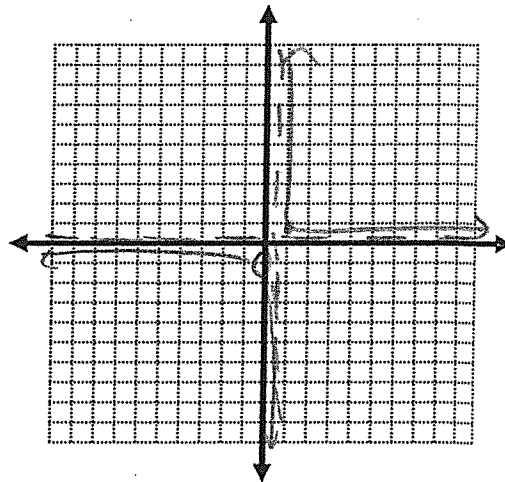
Hole (0, -1) End Behavior _____

as x → ∞, y → 0 / as x → -∞, y → 0

Behavior at each point of discontinuity _____

as x → 0⁻, y → -1 / as x → 0⁺, y → -1

as x → 1/3⁻, y → -∞ / as x → 1/3⁺, y → ∞



$$11. f(x) = \frac{x^2+x-2}{x^2-x-6} = \frac{(x+2)(x-1)}{(x-3)(x+2)}$$

x-int (1, 0) y-int (0, 1/3)

VA x=3 HA/OA y=1

Domain (-∞, -2) ∪ (-2, 3) ∪ (3, ∞) Range (-∞, 3/5) ∪ (3/5, 1) ∪ (1, ∞)

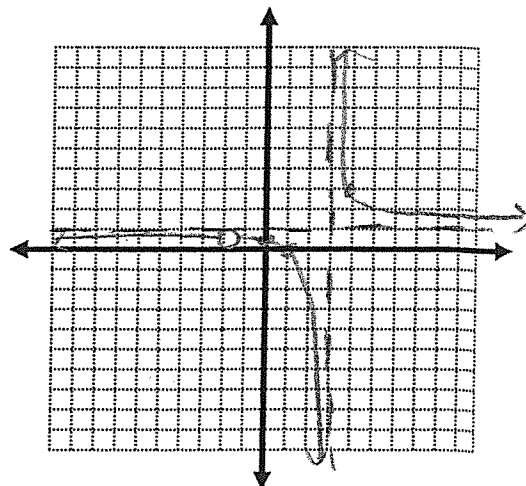
Hole (-2, 3/5) End Behavior _____

as x → ∞, y → 1 / as x → -∞, y → 1

Behavior at each point of discontinuity _____

as x → -2⁻, y → 3/5 / as x → -2⁺, y → 3/5

as x → 3⁻, y → -∞ / as x → 3⁺, y → ∞



$$12. f(x) = \frac{x^2-9}{x^2-2x-3} = \frac{(x+3)(x-3)}{(x-3)(x+1)} = \frac{x+3}{x+1}$$

x-int $(-3, 0)$ y-int $(0, 3)$

VA $x = -1$ HA/OA $y = 1$
 $(-\infty, -1) (-1, 3)$ $(-\infty, 1) (1, 3/2) (3/2, \infty)$
 Domain $x \neq -1$ Range $y \neq 1$

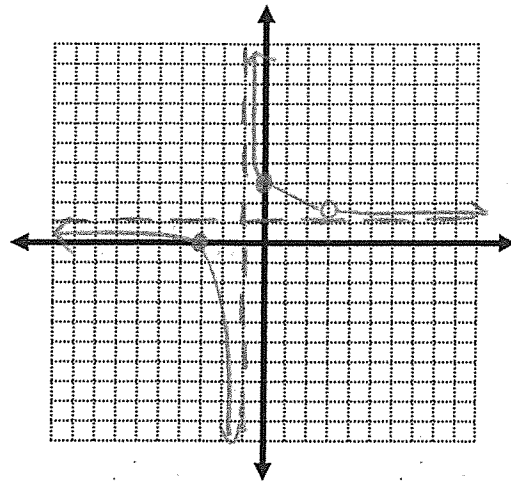
Hole $(3, 3/2)$ End Behavior _____

$as x \rightarrow \infty, y \rightarrow 1$ | $as x \rightarrow -\infty, y \rightarrow 1$

Behavior at each point of discontinuity _____

$as x \rightarrow -1^-, y \rightarrow -\infty$ | $as x \rightarrow -1^+, y \rightarrow \infty$

$as x \rightarrow 3^-, y \rightarrow 3/2$ | $as x \rightarrow 3^+, y \rightarrow 3/2$



$$13. f(x) = \frac{x^2+4x-5}{x}$$

x-int $(-5, 0) (1, 0)$ y-int none

VA $x = 0$ HA/OA $y = x + 4$

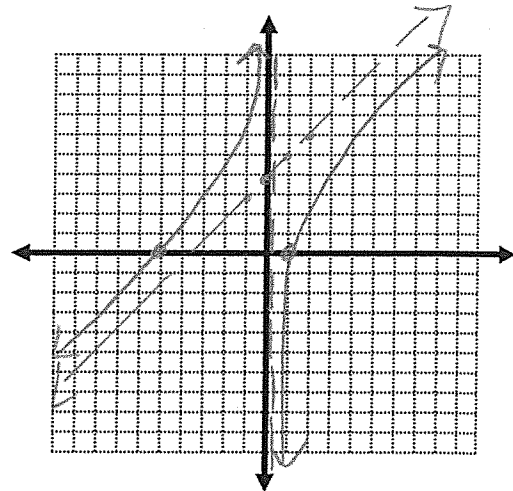
Domain $(-\infty, 0) (0, \infty)$ Range $(-\infty, \infty)$

Hole none End Behavior _____

$as x \rightarrow \infty, y \rightarrow \infty$ | $as x \rightarrow -\infty, y \rightarrow -\infty$

Behavior at each point of discontinuity _____

$as x \rightarrow 0^-, y \rightarrow \infty$ | $as x \rightarrow 0^+, y \rightarrow -\infty$



$$14. f(x) = \frac{x^2+5}{x}$$

x-int none y-int none

VA $x = 0$ HA/OA $y = x$

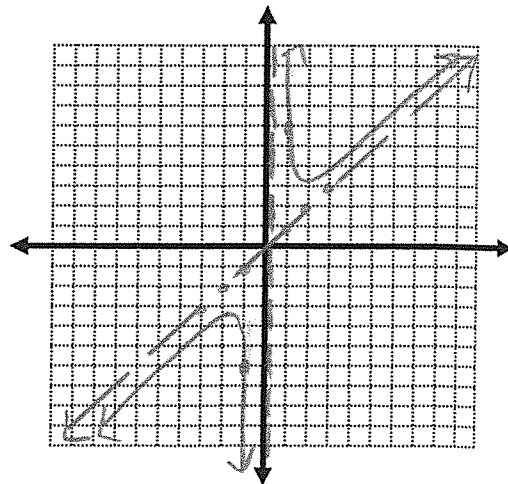
Domain $(-\infty, 0) (0, \infty)$ Range _____

Hole none End Behavior _____

$as x \rightarrow \infty, y \rightarrow \infty$ | $as x \rightarrow -\infty, y \rightarrow -\infty$

Behavior at each point of discontinuity _____

$as x \rightarrow 0^-, y \rightarrow -\infty$ | $as x \rightarrow 0^+, y \rightarrow \infty$



15. $f(x) = \frac{x^2}{3x+1}$

x-int (0,0) y-int (0,0)

VA $x = -1/3$ HA/OA $y = 1/3x - 1/9$

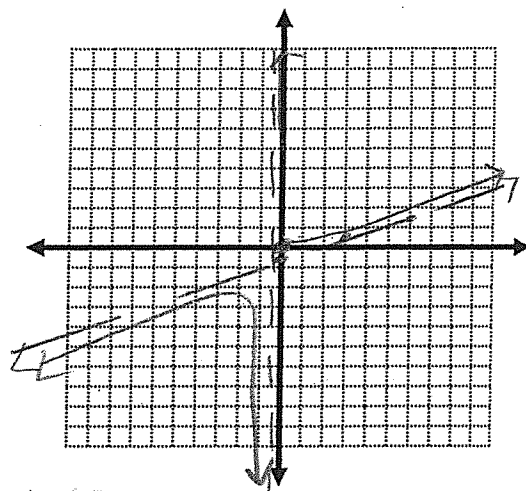
Domain $(-\infty, -1/3) \cup (-1/3, \infty)$ Range _____

Hole none End Behavior _____

$\text{as } x \rightarrow \infty, y \rightarrow \infty$ | $\text{as } x \rightarrow -\infty, y \rightarrow -\infty$

Behavior at each point of discontinuity _____

$\text{as } x \rightarrow -1/3^-, y \rightarrow -\infty$ | $\text{as } x \rightarrow -1/3^+, y \rightarrow \infty$



16. $f(x) = \frac{1-x^2}{x}$

x-int (1,0) (-1,0) y-int none

VA $x = 0$ HA/OA $y = -x$

Domain $(-\infty, 0) \cup (0, \infty)$ Range $(-\infty, \infty)$

Hole none End Behavior _____

$\text{as } x \rightarrow \infty, y \rightarrow -\infty$ | $\text{as } x \rightarrow -\infty, y \rightarrow \infty$

Behavior at each point of discontinuity _____

$\text{as } x \rightarrow 0^-, y \rightarrow -\infty$ | $\text{as } x \rightarrow 0^+, y \rightarrow \infty$

