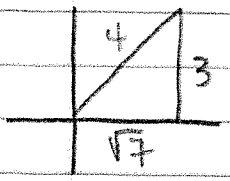
Notes section 5.5 – Double Angles

double angles

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \begin{cases} \cos^2\theta - \sin^2\theta \\ 2\cos^2\theta - 1 \\ 1 - 2\sin^2\theta \end{cases}$

1. If $\sin\theta = 3/4$ on the interval $(0, \pi/2)$ find $\sin 2\theta$ and $\cos 2\theta$



$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2(3/4)(\sqrt{7}/4)$
$= 6\sqrt{7}/16 = \boxed{3\sqrt{7}/8}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= (\sqrt{7}/4)^2 - (3/4)^2$
$= 7/16 - 9/16$
$= -2/16 = \boxed{-1/8}$

$\csc 2\theta = \frac{1}{\sin 2\theta}$

2. Solve $\cos 2\theta = -\sin^2\theta$ for $[0, 2\pi)$

$1 - 2\sin^2\theta = -\sin^2\theta$
$1 = \sin^2\theta$
$\sin\theta = \pm 1$

$\boxed{\theta = \pi/2, 3\pi/2}$

3. solve $\sin^2\theta + \cos 2\theta - \cos\theta = 0$ for $[0, 2\pi)$

$\sin^2\theta + \cos^2\theta - \sin^2\theta - \cos\theta = 0$

$\cos^2\theta - \cos\theta = 0$

$\cos\theta(\cos\theta - 1) = 0$

$\cos\theta = 0 \quad \cos\theta = 1$

$\theta = \pi/2, 3\pi/2, \quad \theta = 0\pi$