

Notes Section 5.4 - Sum and Difference Identities

$$\sin(u+v) = \sin u \cos v + \sin v \cos u$$

$$\sin(u-v) = \sin u \cos v - \sin v \cos u$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

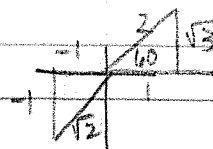
• find the exact value.

$$\begin{aligned} 1. \quad \cos 75^\circ &= \cos(45^\circ + 30^\circ) \\ &= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \end{aligned}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\begin{aligned} 2. \quad \sin \frac{11\pi}{12} &= \sin\left(\frac{5\pi}{4} - \frac{\pi}{3}\right) \\ &= \sin \frac{5\pi}{4} \cos \frac{\pi}{3} - \sin \frac{\pi}{3} \cos \frac{5\pi}{4} \\ &= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) \\ &= \left(-\frac{\sqrt{2}}{4}\right) + \left(\frac{\sqrt{6}}{4}\right) \end{aligned}$$

$$= \frac{-\sqrt{2} + \sqrt{6}}{4}$$



$$\tan x = \frac{\sin x}{\cos x}$$

$$\sec x = \frac{1}{\cos x}$$

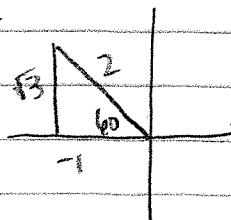
• find the exact value

$$3. \quad \cos \frac{7\pi}{8} \cos \frac{5\pi}{24} + \sin \frac{7\pi}{8} \sin \frac{5\pi}{24}$$

$$\cos\left(\frac{7\pi}{8} - \frac{5\pi}{24}\right)$$

$$\cos\left(\frac{21\pi}{24} - \frac{5\pi}{24}\right)$$

$$\cos\left(\frac{16\pi}{24}\right) = \cos\left(\frac{2\pi}{3}\right) = \boxed{-\frac{1}{2}}$$



$$\begin{aligned} 165^\circ &= \\ 225 - 60 \end{aligned}$$

• simplify

$$4. \sin \pi/3 \cos \pi/4 + \cos \pi/3 \sin \pi/4$$

$$\sin(\pi/3 + \pi/4)$$

$$\sin(4\pi/12 + 3\pi/12)$$

$$\boxed{\sin(7\pi/12)}$$

• verify

$$5. \cos(\pi/2 - x) = \sin x$$

$$\cos \pi/2 \cos x + \sin \pi/2 \sin x = \sin x$$

$$(0) \cos x + (1) \sin x = \sin x$$

$$\sin x = \sin x$$

• solve

$$6. \sin(x + \pi/4) - \sin(x - \pi/4) = 0$$

$$\sin x \cos \pi/4 + \cos x \sin \pi/4 - [\sin x \cos \pi/4 - \cos x \sin \pi/4] = 0$$

$$\cancel{\sin x}(\sqrt{2}/2) + \cos x(\sqrt{2}/2) - \cancel{\sin x}(\sqrt{2}/2) + \cos x(\sqrt{2}/2) = 0$$

$$\sqrt{2}/2 \cos x + \sqrt{2}/2 \cos x = 0$$

$$\sqrt{2} \cos x = 0$$

$$\cos x = 0$$

$$\boxed{x = \pi/2, 3\pi/2}$$