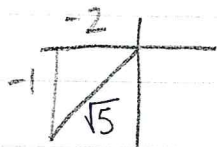
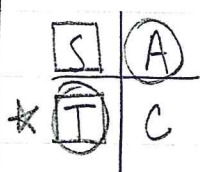


## Notes Section 5.1 - Trig Identities

1. If  $\cot \theta = 2$  and  $\cos \theta < 0$ , find  $\sin \theta$  and  $\cos \theta$ .



$$\sin \theta = -\frac{1}{\sqrt{5}}$$

$$\cos \theta = -\frac{2}{\sqrt{5}}$$

### Reciprocal identities

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

### Quotient identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Pythagorean identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

## cofunction identities

$$\sin \theta = \cos(\pi/2 - \theta)$$

$$\cos \theta = \sin(\pi/2 - \theta)$$

$$\tan \theta = \cot(\pi/2 - \theta)$$

$$\cot \theta = \tan(\pi/2 - \theta)$$

$$\sec \theta = \csc(\pi/2 - \theta)$$

$$\csc \theta = \sec(\pi/2 - \theta)$$

## odd-even identities

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\csc(-\theta) = -\csc \theta$$

$$\sec(-\theta) = \sec \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\cot(-\theta) = -\cot \theta$$

• simplify

$$1. \sqrt{\cos x (1 - \sin^2 x)}$$

$$= \sqrt{\cancel{\cos x} \cdot \cos^2 x}$$

$$= \boxed{\cos x}$$

$$2. \cos x \tan x - \sin x \cos^2 x$$

$$\cancel{\cos x} \cdot \sin x / \cancel{\cos x} - \sin x \cos^2 x$$

$$\sin x - \sin x \cos^2 x$$

$$\sin x (1 - \cos^2 x)$$

$$\sin x (\sin^2 x) = \boxed{\sin^3 x}$$

$$\begin{aligned}
3. \quad & \frac{\sec x}{1 - \sec x} - \frac{\sec x}{1 + \sec x} \\
= & \frac{\sec x(1 + \sec x)}{1 - \sec^2 x} - \frac{\sec x(1 - \sec x)}{1 - \sec^2 x} \\
= & \frac{\sec x + \sec^2 x - \sec x + \sec^2 x}{1 - \sec^2 x} \\
= & \frac{2\sec^2 x}{1 - \sec^2 x} \\
= & \frac{2\sec^2 x}{\tan^2 x} \\
= & \frac{2}{\cancel{\cos^2 x}} \cdot \frac{\cancel{\cos^2 x}}{\sin^2 x} \\
= & \frac{2}{\sin^2 x} = \boxed{2 \csc^2 x}
\end{aligned}$$

$$\begin{aligned}
4. \quad & \frac{1 + \tan^2 x}{\csc^2 x} \quad [\text{rewrite so not a fraction}] \\
= & \frac{\sec^2 x}{\csc^2 x} \\
= & \frac{1}{\cos^2 x} \cdot \frac{\sin^2 x}{1} \\
= & \frac{\sin^2 x}{\cos^2 x} \\
= & \boxed{\tan^2 x}
\end{aligned}$$

5.  $\frac{\cos^2 x}{1 - \sin x}$  [rewrite so not a fraction]

$$= \frac{1 - \sin^2 x}{1 - \sin x}$$

$$= \frac{(1 + \sin x)(1 - \sin x)}{1 - \sin x}$$

$$= \frac{(1 + \sin x)\cancel{(1 - \sin x)}}{\cancel{1 - \sin x}}$$

$$= 1 + \sin x$$

6.  $\frac{4}{\sec x - \tan x}$

$$= \frac{4}{\sec x - \tan x} \cdot \frac{\sec x + \tan x}{\sec x + \tan x}$$

$$= \frac{4(\sec x + \tan x)}{\sec^2 x - \tan^2 x}$$

$$= 4(\sec x + \tan x)$$