7.3 Use Similar Right Triangles

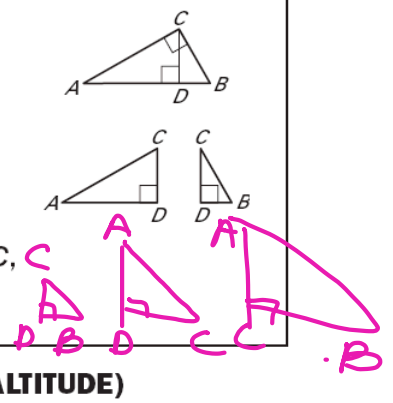
Goal • Use properties of the altitude of a right triangle.

similar ~

THEOREM 7.5

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are ~ to the original triangle and to each other.

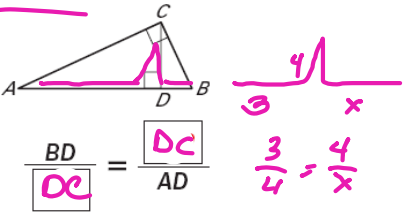
△CBD ~ △ABC, △ACD ~ △ABC, and △CBD ~ △ACD.



THEOREM 7.6: GEOMETRIC MEAN (ALTITUDE) THEOREM

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.

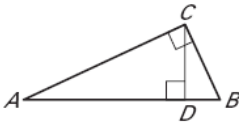


$$\frac{BD}{DC} = \frac{DC}{AD} \qquad \frac{3}{4} = \frac{4}{x}$$

THEOREM 7.7: GEOMETRIC MEAN (LEG) THEOREM

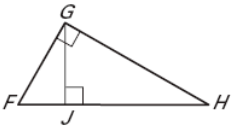
In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.



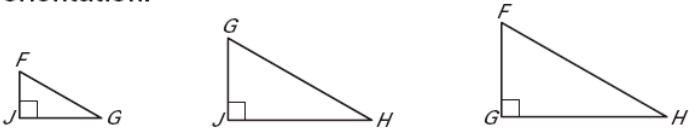
$$\frac{AB}{CB} = \frac{CB}{DB} \text{ and } \frac{AB}{AC} = \frac{AC}{AD}$$

Identify the similar triangles in the diagram.



Solution

Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.

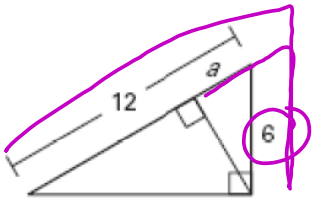


Summary

△ FJG ~ △ GJH ~ △ FGH

7.3 Use Similar Right Triangles

4.

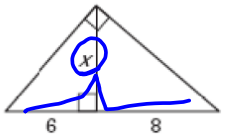


$\frac{12}{6} = \frac{6}{a}$

$12a = 36$

$a = 3$

5.

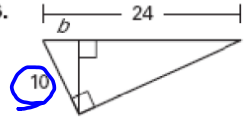


$\frac{6}{x} = \frac{x}{8}$

$x^2 = 48$

6.93

6.

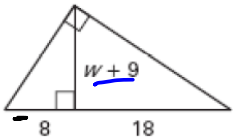


$\frac{24}{10} = \frac{10}{b}$

$24b = 100$

$b = 4.17$

7.



$\frac{8}{(w+9)} = \frac{(w+9)}{18}$

$144 = w^2 + 18w + 81$

$0 = w^2 + 18w - 63$

$0 = (w+21)(w-3)$

$w = -21$ $\quad w = 3$

63

$-3 + 21$

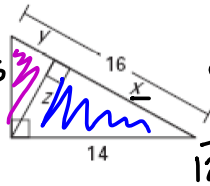
$w + 21 = 0$ $\quad w - 3 = 0$

$w = -21$ $\quad w = 3$

$16^2 - 14^2$

7.75

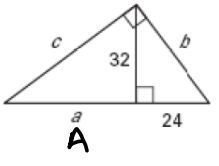
8.



$z = 6.78$

$\frac{z}{12.25} = \frac{7.75}{14}$

9.



A

	sl	ll	hyp
sm Δ	y	z	7.75
m Δ	z	x	14
l Δ	7.75	14	16

$\frac{y}{7.75} = \frac{7.75}{16}$

$y = 3.75$

$x = 12.25$

	sl	ll	H
s Δ	24	32	b
m Δ	32	A	c
l Δ	b	c	a+24

$\frac{24}{32} = \frac{32}{A} = 42.67$

$\frac{42.67}{c} = \frac{c}{66.67}$

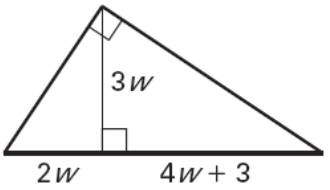
$c = 53.34$

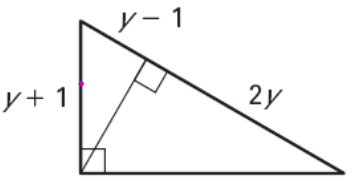
$\frac{24}{b} = \frac{b}{66.67}$

$b = 40.00$

Summary

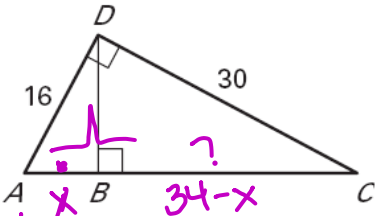
7.3 Use Similar Right Triangles





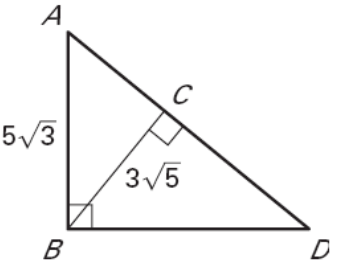
Use the Geometric Mean Theorems to find *AC* and *BD*.

13.



$\frac{x}{16} = \frac{16}{34}$ 34

14.



7.3

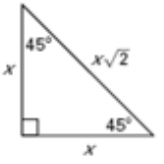
7.4 Special Right Triangles

Goal • Use the relationships among the sides in special right triangles.

x x x√2

THEOREM 7.8: 45°-45°-90° TRIANGLE THEOREM

In a 45°-45°-90° triangle, the hypotenuse is √2 times as long as each leg.

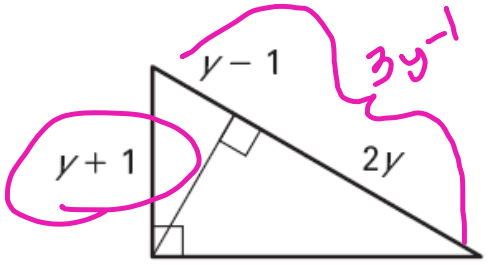


hypotenuse = leg • √2

Leg= hyp ÷ √2

* $\frac{h\sqrt{2}}{2}$

Summary



$$\frac{y-1}{y+1} = \frac{y+1}{3y-1}$$

$$y^2+2y+1 = 3y^2-4y+1$$
$$-y^2 \qquad\qquad -y^2$$

$$2y+1 = 2y^2-4y+1$$
$$-2y \qquad\qquad -2y$$

$$1 = 2y^2-6y+1$$
$$-1 \qquad\qquad -1$$

$$0 = 2y^2-6y$$

	y	-1
$3y$	$3y^2$	$-3y$
-1	$-y$	$+1$

	y	$+1$
y	y^2	$1y$
$+1$	$1y$	1

$$0 = 2y^2-6y$$

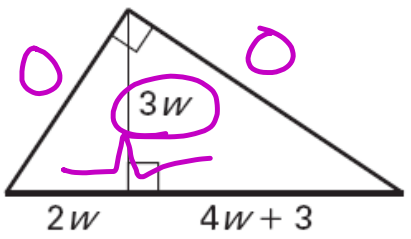
$$0 = 2y(y-3)$$

$$0, 3$$

$$6y = 2y^2$$

$$3y = y^2$$

$$3 = y \qquad y = 0$$



$$\frac{2w}{3w} = \frac{3w}{4w+3}$$

$$9w^2 = 2w(4w+3)$$

$$9w^2 = 8w^2 + 6w$$

$$w^2 = 6w$$

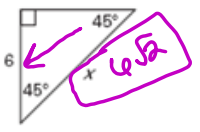
$$\frac{w}{w} \quad \frac{w}{}$$

$w = 6$ $\qquad$ $w = 0$

7.4 Special Right Triangles

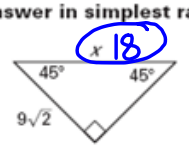
Using 45, 45, 90 Triangle Rule

Find the value of x. Write your answer in simplest radical form.

1. 

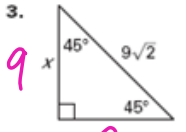
$6\sqrt{2}$

45 45 90
x x $x\sqrt{2}$
6 6 $6\sqrt{2}$

2. 

18

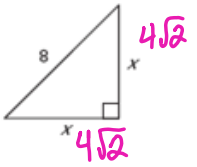
45 45 90
x x $x\sqrt{2}$
$9\sqrt{2}$ $9\sqrt{2}$ $9\sqrt{2}(\sqrt{2})$
18

3. 

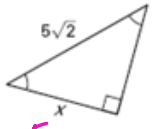
9

4. 

$12\sqrt{2}$

5. 

$4\sqrt{2}$
$4\sqrt{2}$

6. 

5

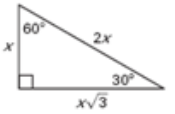
30, 60, 90 Triangle

THEOREM 7.9: 30°-60°-90° TRIANGLE THEOREM

In a 30°-60°-90° triangle, the hypotenuse is $2x$'s as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

hypotenuse = 2 • shorter leg

longer leg = shorter leg • $\sqrt{3}$

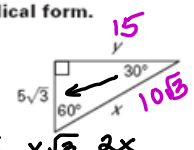


Shorter leg= hyp ÷ 2 or

long leg ÷ $\sqrt{3}$ ← * $\frac{l\sqrt{3}}{3}$

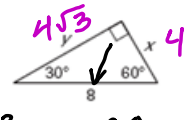
Using 30, 60, 90 Triangle Rule

Find the value of each variable. Write your answers in simplest radical form.

7. 

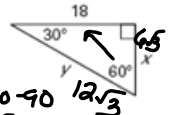
15
$10\sqrt{3}$

x $x\sqrt{3}$ 2x
30-60-90
$5\sqrt{3}$ → 15 $10\sqrt{3}$

8. 

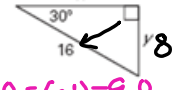
$4\sqrt{3}$
4

30-60-90
x $x\sqrt{3}$ 2x
4 → $4\sqrt{3}$ 8

9. 

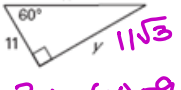
$12\sqrt{3}$

30-60-90
x $x\sqrt{3}$ 2x
$6\sqrt{3}$ ← 18 $12\sqrt{3}$

10. 

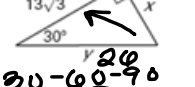
$8\sqrt{3}$
8

30-60-90
x $x\sqrt{3}$ 2x
8 → $8\sqrt{3}$ 16

11. 

22
$11\sqrt{3}$

30-60-90
x $x\sqrt{3}$ 2x
11 $11\sqrt{3}$ 22

12. 

13
24

30-60-90
x $x\sqrt{3}$ 2x
13 ← $13\sqrt{3}$ 26

Summary

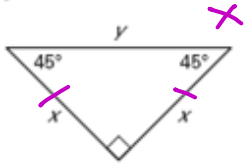
$\frac{13\sqrt{3}}{\sqrt{3}}$

7.4 Special Right Triangles

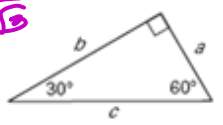
Complete the table.

45 45 90
x x x√2

13.



14.



45 x
90 x√2

x	5	4	√2	9	12√2
y	5√2	4√2	2	9√2	24

√2 √2

30 x
60 x√3
90 2x

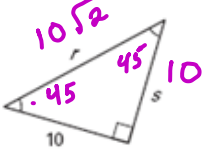
a	9	3√3	5	11	8
b	9√3	9	5√3	11√3	8√3
c	18	6√3	10	22	16

$\frac{24}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$

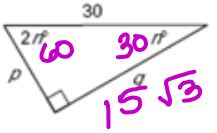
$\frac{16}{2} = \frac{2x}{2}$

Find the value of each variable. Write your answers in simplest radical form.

15.

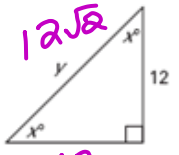


16.



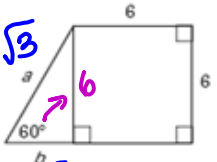
15 15√3

17.



12√2

18.

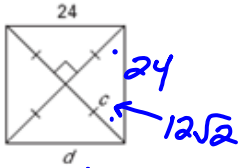


4√3
6
2√3

30 60 90
x x√3 2x
2√3 6 4√3

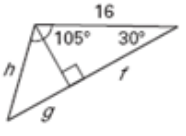
$\frac{6}{\sqrt{3}}$

19.



24
12√2
24

20.



The side lengths of a triangle are given. Determine whether it is a *45°-45°-90° triangle*, a *30°-60°-90° triangle*, or *neither*.

21. 5, 10, $5\sqrt{3}$

30-60-90

22. 7, 7, $7\sqrt{2}$

Neither

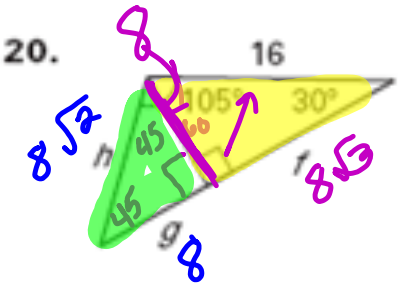
23. 6, 6, $6\sqrt{2}$

45-45-90

7.4

Summary

20.



105
- 60
45

30-60-90
x√3 2x
x 16
8 → 8√3

45 45 90
x x x√2
8 8 8√2